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NAVAL SURFACE WEAPONS CENTER DAHLGREN LAB VA
BEACON TRACKING SYSTEMS SIMULATION.(U)
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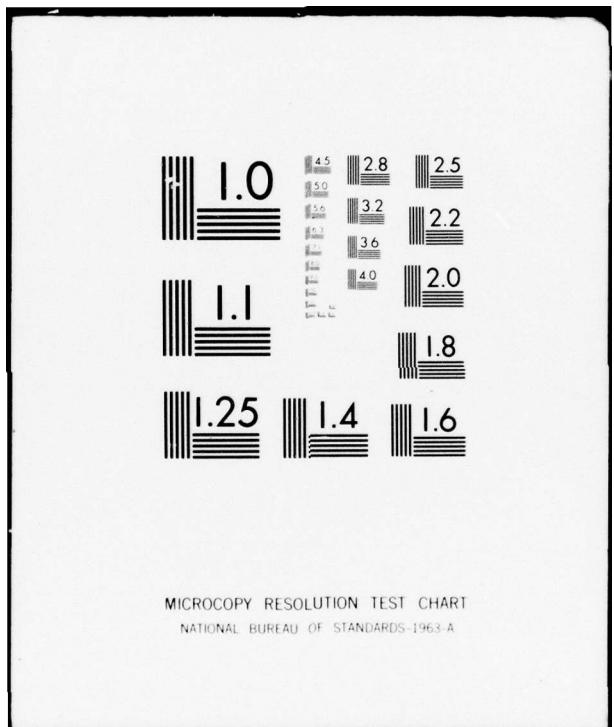
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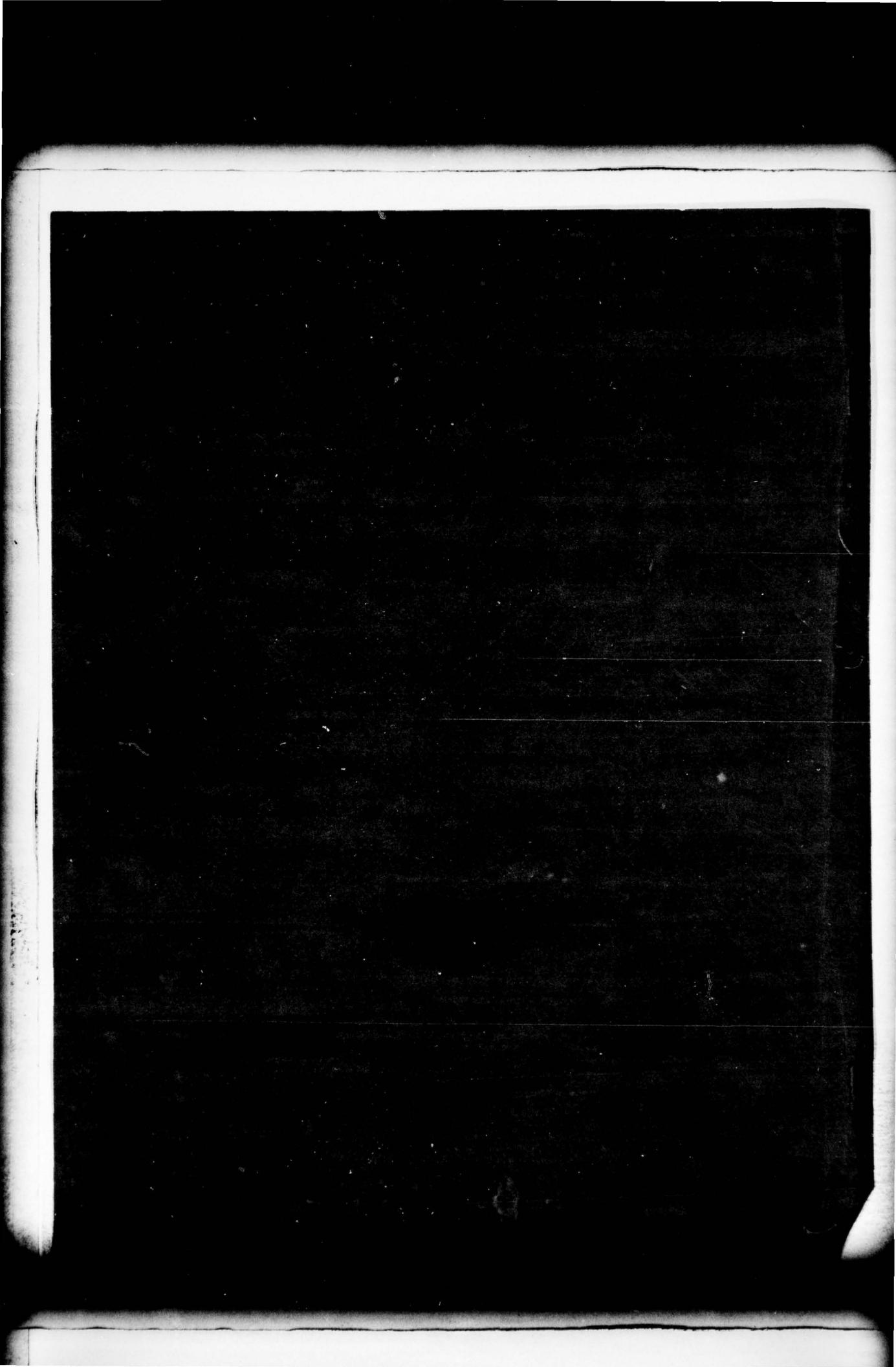
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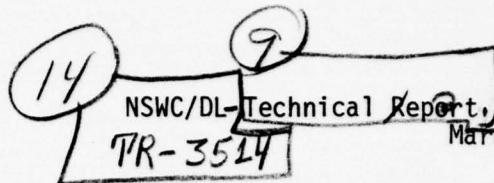
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(9) BEACON TRACKING SYSTEMS SIMULATION.

(10) Elodie S. Colquitt
Warfare Analysis Department

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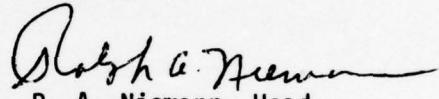
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FOREWORD

This report describes results from computer simulations performed to predict the effects of various parameters on the Beacon Tracking System. The results were used to plan the implementation of the Beacon Tracking System on the Naval Surface Weapons Center/Dahlgren Laboratory Test Range. Ted Sims of DK-11 assisted in the formulation of the computer simulation program. This report has been reviewed by R. J. Anderle, Head, Astronautics & Geodesy Division.

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Warfare Analysis Department

CONTENTS

	<u>Page</u>
FOREWORD	i
INTRODUCTION	1
SIMULATION	
1. TDOA Concept	1
2. Receiver Geometry.	2
3. Coordinate System.	2
4. Data	3
RESULTS	
1. Data Noise	3
2. Target Maneuvers	4
3. Receiver Positions	4
4. Receiver Failure	6
APPENDICES	
A. Figures.	A
B. Method of Least Squares.	B
DISTRIBUTION	

INTRODUCTION

As part of the research and development effort in producing the hardware and software for the Beacon Tracking System, a simulation of the data and the data reduction scheme was performed to investigate the sensitivity of the system to certain key parameters. The simulation consists of generating synthetic TDOA data for a given test set up, and performing a weighted least squares solution of this data for target position. Target maneuver data such as might be encountered in a true tracking situation is included in the simulation. Each of the features of the simulation is discussed in detail below.

SIMULATION

1. TDOA Concept

The TDOA concept is based on the principle that a range difference from a spherical wave front, centered at an emitter, to two known receivers will define a hyperboloid passing through the emitter. Simultaneous solution of three hyperboloids gives three dimensional coordinates of the emitter. Three hyperboloids are defined if a minimum of four receivers (three sets of two) make range difference measurements. Additional sets over-define the solution permitting investigation of additional parameters.

The TDOA system studied is shown in Figure 1. A transmitter or beacon emits a pulse that travels to a master receiver and several slave transceivers placed at known locations. Upon receipt of the target beacon each slave transmits a pulse back to the master. T_M and T_1 are the times the target pulse travels from the beacon to the master and a slave, respectively. T_{1M} is the time a pulse travels from the slave to the master. ΔT_1 is the slave transceiver reaction time (hereafter called receiver thru time or RTT). Then,

$$TDOA_1 = (T_1 + \Delta T_1 + T_{1M}) - T_M \quad (1)$$

Since the locations of the slave and master stations are known, T_{1M} is known. It is the distance between them divided by the speed of light. Receiver thru time is hardware dependent and empirically determined. Removing the two known quantities leaves

$$TDOA_1 = T_1 - T_M \quad (2)$$

If X_M , Y_M , Z_M are the coordinates of the master receiver; X_n , Y_n , Z_n are the coordinates of the n slave receivers, and X , Y , Z are the unknown beacon position,

$$T_M = \frac{1}{C} [(X_M - X)^2 + (Y_M - Y)^2 + (Z_M - Z)^2]^{1/2} \quad (3)$$

$$T_1 = \frac{1}{C} [(X_1 - X)^2 + (Y_1 - Y)^2 + (Z_1 - Z)^2]^{1/2} \quad (4)$$

⋮
⋮

$$T_n = \frac{1}{C} [(X_n - X)^2 + (Y_n - Y)^2 + (Z_n - Z)^2]^{1/2} \quad (5)$$

For $(n+1)$ receivers (1 master, n slaves), n independent TDOA observations are solved simultaneously for X , Y , and Z .

The n sets of simultaneous equations are solved by the method of weighted least squares discussed in Appendix B.

2. Receiver Geometry

The Beacon Tracking System operates in the geographic area shown on the map in Figure 2. It is designed to track over the range from 4 to 24 km. The master station is located at the main firing range at NSWC/DL. Slave stations are located along the shores of the Potomac River, down-range from NSWC/DL. Three criterion are met in the selection of each of the slave sites: (1) the slave station has a clear line of sight to the master station for data telemetry; (2) the geographical survey position of the site is known accurately, and (3) the site is easily accessible to NSWC/DL personnel. On the basis of these criterion the following positions are chosen for study in the simulation: station 5, 9, 25, 29, 12 (also called Swan Point), Cobb Island, and Oakland, an inland point on the Virginia side of the river (not visible in Figure 2).

3. Coordinate System

For the Beacon Tracking simulation the various slave sites are referenced to the master station in cartesian coordinates X , Y , Z . The Master station is at 0,0 with positive X "East", positive Y "North", and positive Z "Up". All the sites are surveyed relative to a reference ellipsoid, then transformed from geodetic to cartesian coordinates with the master station as origin.

4. Data

Three different kinds of flight paths are used as input to the tracking simulation: (1) a straight line path up the river toward the master station at a velocity of about 450 mph and altitude of one kilometer simulates a nominal aircraft flight path (this path is called a "riverflight" for reference); (2) an aircraft flight path as in (1) but with a 4 km period sine curve on the X, Y, and Z components simulates aircraft maneuvers (this path is called a "riversincurve"); and (3) a straight line path up river toward the master station, velocity about 250 mph, with an abrupt drop in altitude from 8000 ft. to 4000 ft. simulates the path an inbound missile might take. This path is called a "missile trajectory". Using these three flight paths, synthetic TDOA data is generated for use in the computer simulation.

Sigma on the data class is taken to be $\sigma_D = 1 \times 10^{-9}$. A-priori estimate of the hardware (i.e. receiver) sigmas is taken to be $\sigma_{RTT} = 5 \times 10^{-9}$. Atmospheric refraction effects were not considered to maintain simplicity.

RESULTS

1. Data Noise

To test the effect of data noise on the least squares solution, a comparison is made between two "riverflights", identical except for noise on the synthetic data. The receiver configuration is Master, 12, 9, 25, and 29. Figure 3 shows the trace of the sigma levels in X, Y, and Z (sigmas) over the path from 32 km to 2 km with no noise on the data. A second "riverflight" trajectory is run identical to the first except that 0 to +5 nanoseconds of random noise is put on the data of the second trajectory. Figure 4 shows the difference between the true range to the aircraft and the range computed using the noisy data. The differences in range shown in Figure 4 appear significant because the solution is based on a single data burst at each point. The differences can be greatly reduced by obtaining enough data to permit an "average" of several bursts of data to be taken at each point.

Note that in both Figures 3 and 4 an abrupt increase in solution error is seen near a range of 26 km. This range is the point where the aircraft enters an unfavorable geometry spot in the receiver configuration.

2. Target Maneuvers

To test the ability of the Beacon Tracking System to follow a maneuvering target, a "riversincurve" path is flown using the receiver configuration M, 12, 9, 25, 29. Synthetic data based on these factors were generated. Figures 5 and 6 show the effect the 4 km period sine curves on X, Y, and Z have on the solution. The solution for X and Y (Figure 5) show no wavering over the area of interest, 4 to 24 km. The sine path is seen in the solution for Z (Figure 6); however, acceptable sigma levels are maintained. Figures 7, 8, and 9 show the displacement in X, Y, and Z, respectively, between the "true" and computed positions based on synthetic data.

The reader will note that in nearly all cases presented here, sigma levels in the solution for Z differ from X and Y by an order of magnitude. This difference is geometry related in that the receiver stations all lie in approximately the same plane. This "weakness" in the solution for Z would not be seen were it physically possible to position the receivers in three dimensions around the target flight path.

Figure 10 shows results from a case that is a duplicate of the above "riversincurve" case (Figures 5 and 6) with one small change. In Figure 10 the least squares solution is computed using five iterations instead of three as in the previous case. No effect is demonstrated from the five iteration case over three iterations. Thus, all other cases are solved using three iterations.

To test the ability of the Beacon Tracking System to track the path of an incoming missile, a "missile trajectory" is flown through the configuration M, 12, 9, 25, 29. The target travels at 250 mph at an altitude of 8,000 ft. until it reaches a range of 13 km. Then it rapidly drops to an altitude of 4,000 ft. Synthetic data is generated from these conditions. Figure 11 shows no drastic degradation in the solution for X and Y. Figure 12 shows a slight degradation in the solution for Z at the altitude drop, but with quick recovery. Figures 13, 14, and 15 show the displacement between the "true" and the computed values for target position X, Y, and Z, respectively. No loss of accuracy is seen in X and Y; barely perceptible accuracy changes are seen in Z.

3. Receiver Positions

Our criterion that the receiver sites be easily accessible to NSWC/DL personnel make it desirable to have slave sites on the Virginia side of the Potomac River only. To test the effect on the system

solution if no slave receivers are used on the Maryland side, two cases are given below for two different configurations.

In the first case where no Maryland receivers were used a "rivers-incurve" is flown using receiver configuration M, 9, 25, 29. Synthetic data were generated with these conditions. Figures 16 thru 20 show the solution sigma levels and the displacements in X, Y, and Z between "true" and computed positions. The solution for X in this configuration is acceptable over the range of interest, 4 to 24 km. The solutions for Y and Z, however, are not. Both Y and Z show poor accuracy level over the range from 16 to 24 km. In addition Y and Z solutions degrade significantly around 23 km. This degradation is attributed to the receiver geometry. Because of poor solution confidence in two dimensions and the solution degrading within our area of interest, this configuration is judged inadequate for Beacon Tracking.

The second case where no Maryland receivers were used is a "missile trajectory" with receiver configuration M, 5, 9, 25. Synthetic data were generated with these conditions. Figures 21 thru 25 show the solution confidence levels and the X, Y, Z displacements between "true" and computed target positions. Substituting station 5 for station 29 eliminates the solution degradation due to receiver geometry within our range of interest. The solution is acceptable in X and Y for this configuration, but questionable in Z.

We add two receivers in Maryland to the above case to test solution improvement. The configuration is now M, 5, 9, 25, 12, Cobb. Figures 26 thru 30 show the new solution based on data from this new configuration with the Maryland sites added. Over the range of interest solution improvement is dramatic. In X and Y solution accuracy improves by a factor of two or more. In Z, however, solution accuracy improves by a factor of four or more. Improvement in Z is particularly seen in the range from 14 km to 24 km. Where the Z solution is questionable in this range for the no Maryland receivers case, it is clearly acceptable with the Maryland receivers added.

It is recognized that two factors affect the solution improvement shown in the above case: (1) the addition of two more receivers to the configuration, and (2) the positioning of receivers "around" the target. Other NSWC analysis has demonstrated that the second factor is the most significant. This configuration, M, 5, 9, 25, 12, Cobb is the one selected for the operational Beacon Tracking System.

4. Receiver Failure

At the time of this simulation study the Beacon Tracking System hardware was experimental and its reliability as yet unproven. Therefore, the possibility existed that one or more slave receivers would not function during the course of a tracking sequence. To test the sensitivity of the system solution to the loss of a receiver, two such scenarios are simulated.

In the first a "riversincurve" is flown with receiver net M,9,25, 12, Cobb, Oakland. At each data point each receiver is alternately dropped and the solution performed without that receiver's data contributing to the solution. Figures 31 and 32 show the resultant plots of sigma levels for X,Y and Z, respectively. For this configuration stations 9 and 25 are shown to be the most critical to the solution confidence. Station 9 is particularly critical in all three dimensions. Without data from Station 9, the solution degrades drastically in the range from 12 km to 24 km. Station 25 is critical to the solution for X and Y in the distant ranges (24+ km). The other three stations, Oakland, Swan, and Cobb, are of about equal but lesser importance in X and Y. Station 9 is the critical one in the solution for Z, with 25, Swan, Oakland, and Cobb of about equal but far lesser importance.

In the second scenario a "riversincurve" is flown with receiver configuration M, 5, 9, 25, 12, Cobb. As the geometry suggests, station 25 is again the most critical to the solution in all three dimensions, particularly at the distant ranges. For X and Y (see Figure 33) the other four slave stations are of about equal but far lesser importance. Of particular interest for this scenario is the importance of the various slaves in the solution for Z, as shown in Figure 34. Again, station 25 is the most critical to the solution. However, stations 9 and 12 (Swan) are significantly more important to the Z solution than are stations 5 and Cobb. This reconfirms our earlier observations about the importance of at least one slave site on the Maryland side of the river.

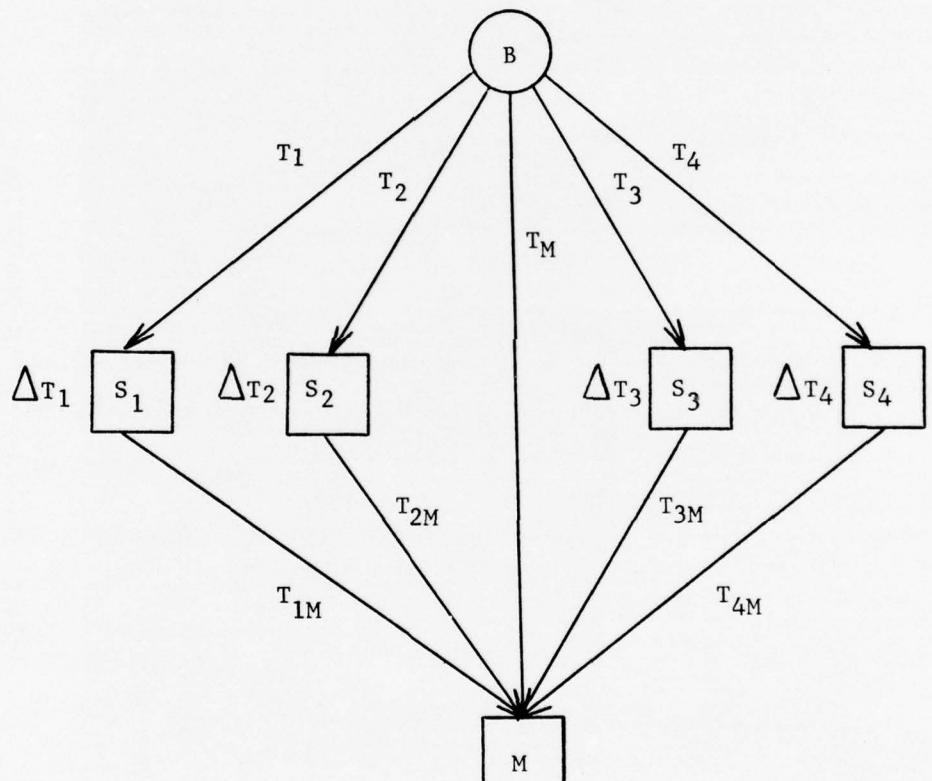
APPENDIX A

FIGURES

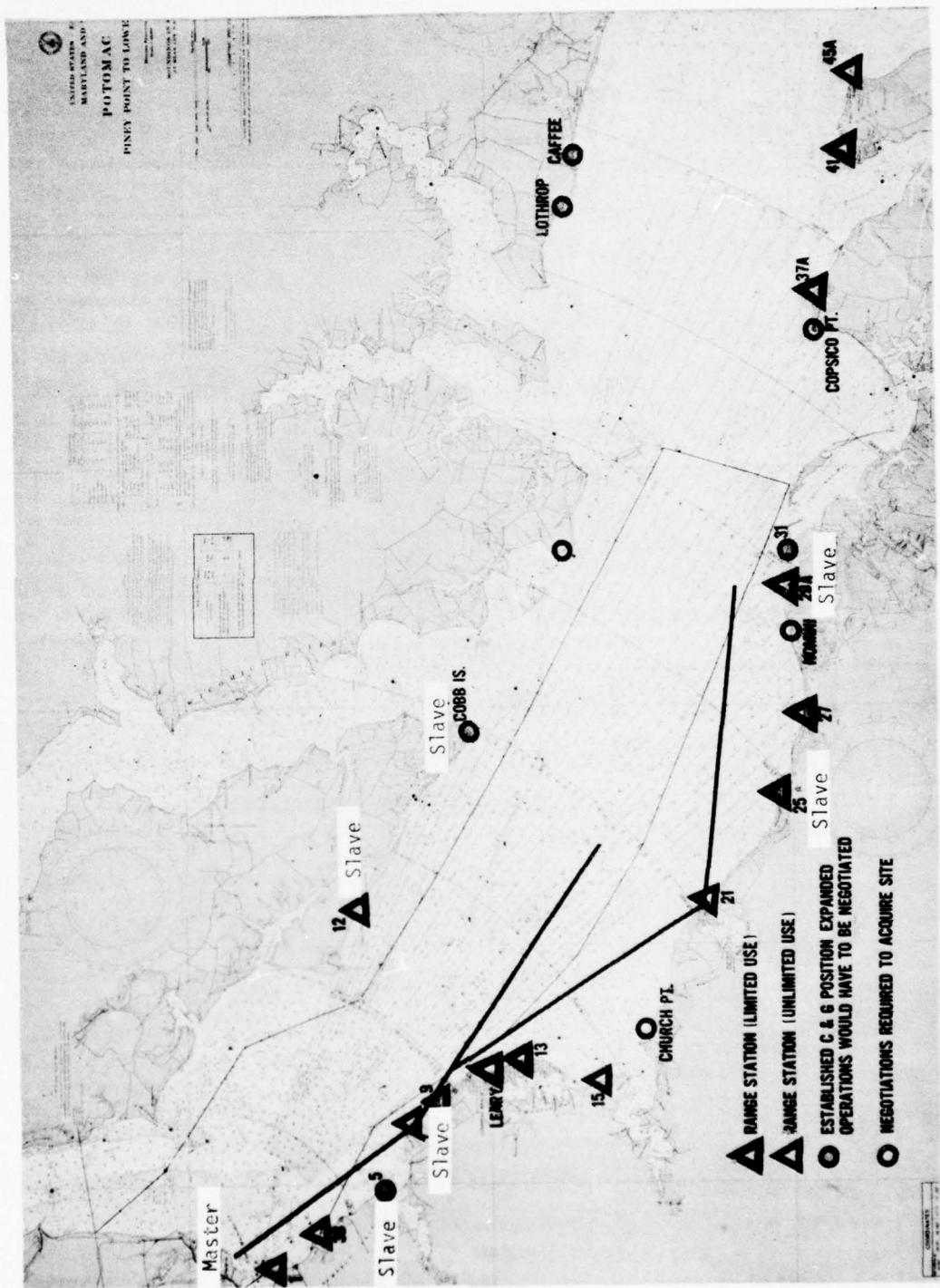
LIST OF FIGURES

<u>Figure</u>	<u>Description</u>	<u>Page</u>
1	TDOA Concept	A-1
2	Beacon Tracking Range Map	A-2
3	X, Y, Z Sigma vs. Range, Straight Line Path	A-3
4	Delta Range vs. Range, Straight Line Path with Noise	A-4
5	X and Y Sigma vs. Range, Sine Path	A-5
6	Z Sigma vs. Range, Sine Path	A-6
7	X Displacement vs. Range, Sine Path	A-7
8	Y Displacement vs. Range, Sine Path	A-8
9	Z Displacement vs. Range, Sine path	A-9
10	X, Y, Z Sigma vs. Range, Sine Path, 5 Iterations	A-10
11	X and Y Sigma vs. Range, Missile Path	A-11
12	Z Sigma vs Range, Missile Path	A-12
13	X Displacement vs. Range, Missile Path	A-13
14	Y Displacement vs. Range, Missile Path	A-14
15	Z Displacement vs. Range, Missile Path	A-15
16	X and Y Sigma vs. Range, Sine Path, No Maryland	A-16
17	Z Sigma vs. Range, Sine Path, No Maryland	A-17
18	X Displacement vs. Range, Sine Path, No Maryland	A-18
19	Y Displacement vs. Range, Sine Path, No Maryland	A-19
20	Z Displacement vs. Range, Sine Path, No Maryland	A-20
21	X and Y Sigma vs. Range, Missile Path, No Maryland	A-21
22	Z Sigma vs. Range, Missile Path, No Maryland	A-22
23	X Displacement vs. Range, Missile Path, No Maryland	A-23
24	Y Displacement vs. Range, Missile Path, No Maryland	A-24
25	Z Displacement vs. Range, Missile Path, No Maryland	A-25
26	X and Y Sigma vs. Range, Missile Path, Maryland Sites	A-26
27	Z Sigma vs. Range, Missile Path, Maryland Sites	A-27
28	X Displacement vs. Range, Missile Path, Maryland Sites	A-28
29	Y Displacement vs. Range, Missile Path, Maryland Sites	A-29
30	Z Displacement vs. Range, Missile Path, Maryland Sites	A-30
31	X and Y Sigma vs. Range, One Slave Out (M,9,25,12,C,0)	A-31
32	Z Sigma vs. Range, One Slave Out (M,9,25,12,C,0)	A-32
33	X and Y Sigma vs. Range, One Slave Out (M,5,9,25,12,C)	A-33
34	Z Sigma vs. Range, One Slave Out (M,5,9,25,12,C)	A-34

FIGURE 1
TDOA CONCEPT



$$TDOA_n = (T_n + \Delta T_n + T_{nM}) - T_M$$



SIGMA VRS RANGE

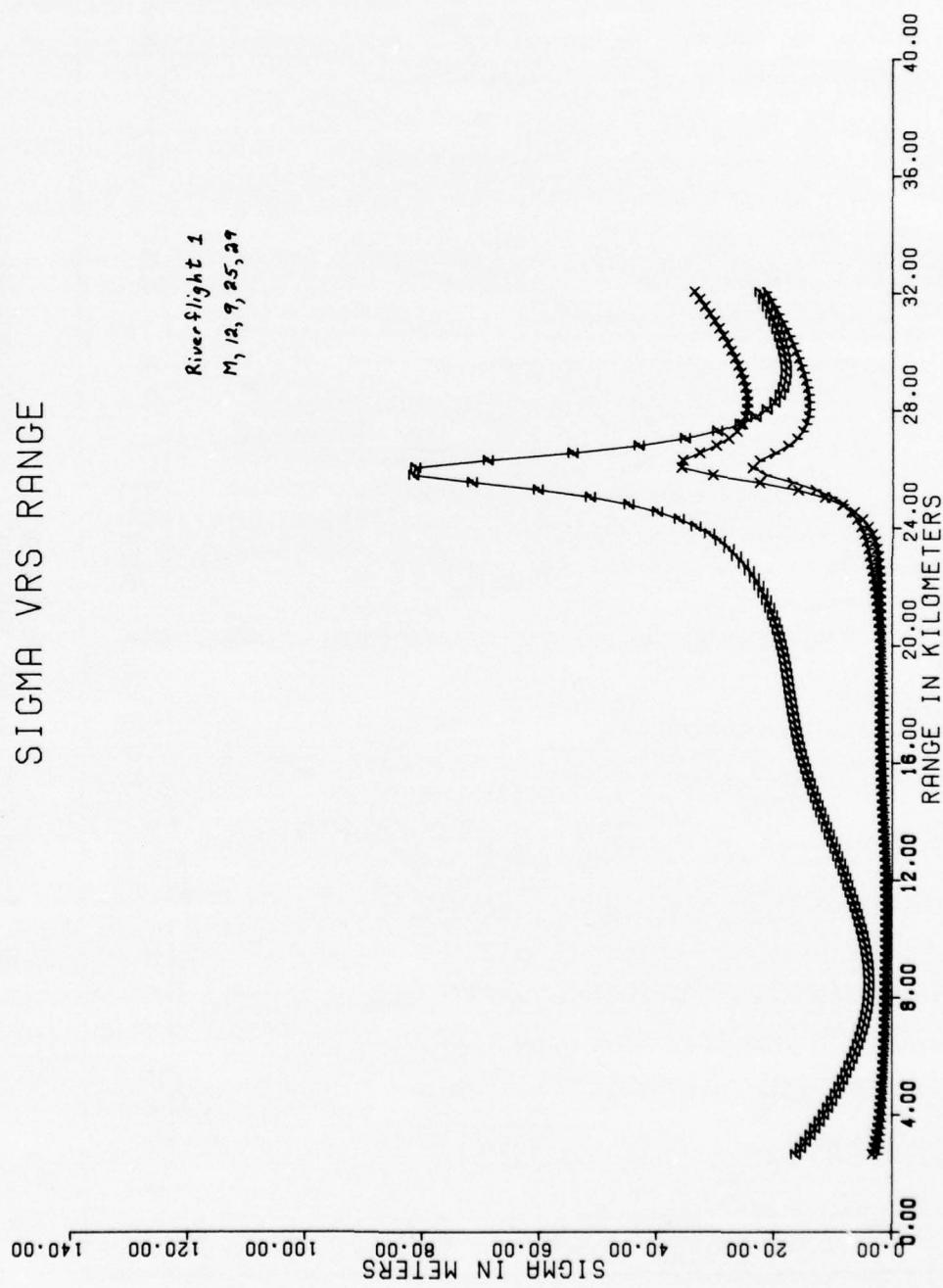
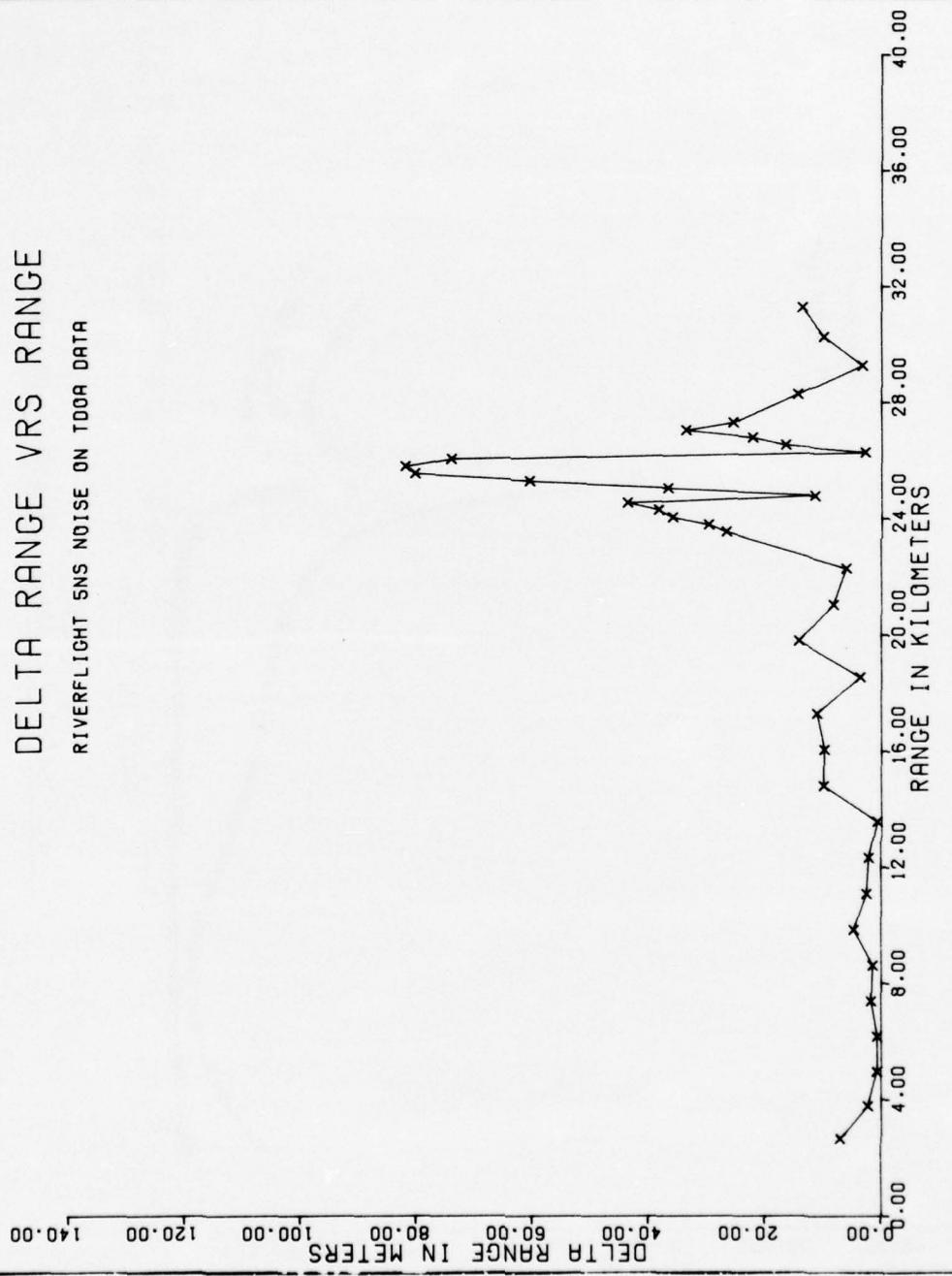


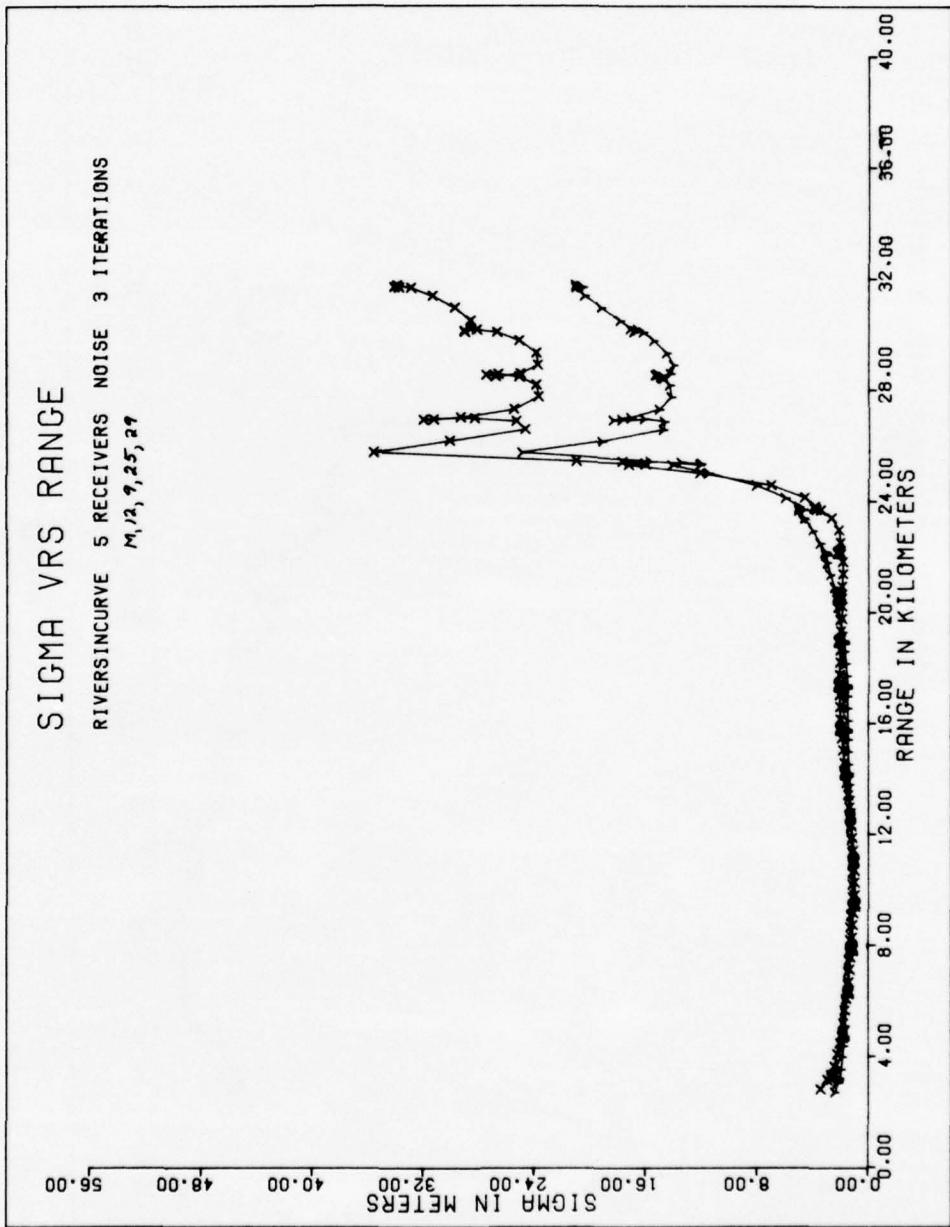
FIGURE 3
X, Y, Z SIGMA VS. RANGE, STRAIGHT LINE PATH

DELTA RANGE VRS RANGE
RIVERFLIGHT SNS NOISE ON TDOA DATA



A-4

FIGURE 4
DELTA RANGE VS. RANGE, STRAIGHT LINE PATH WITH NOISE



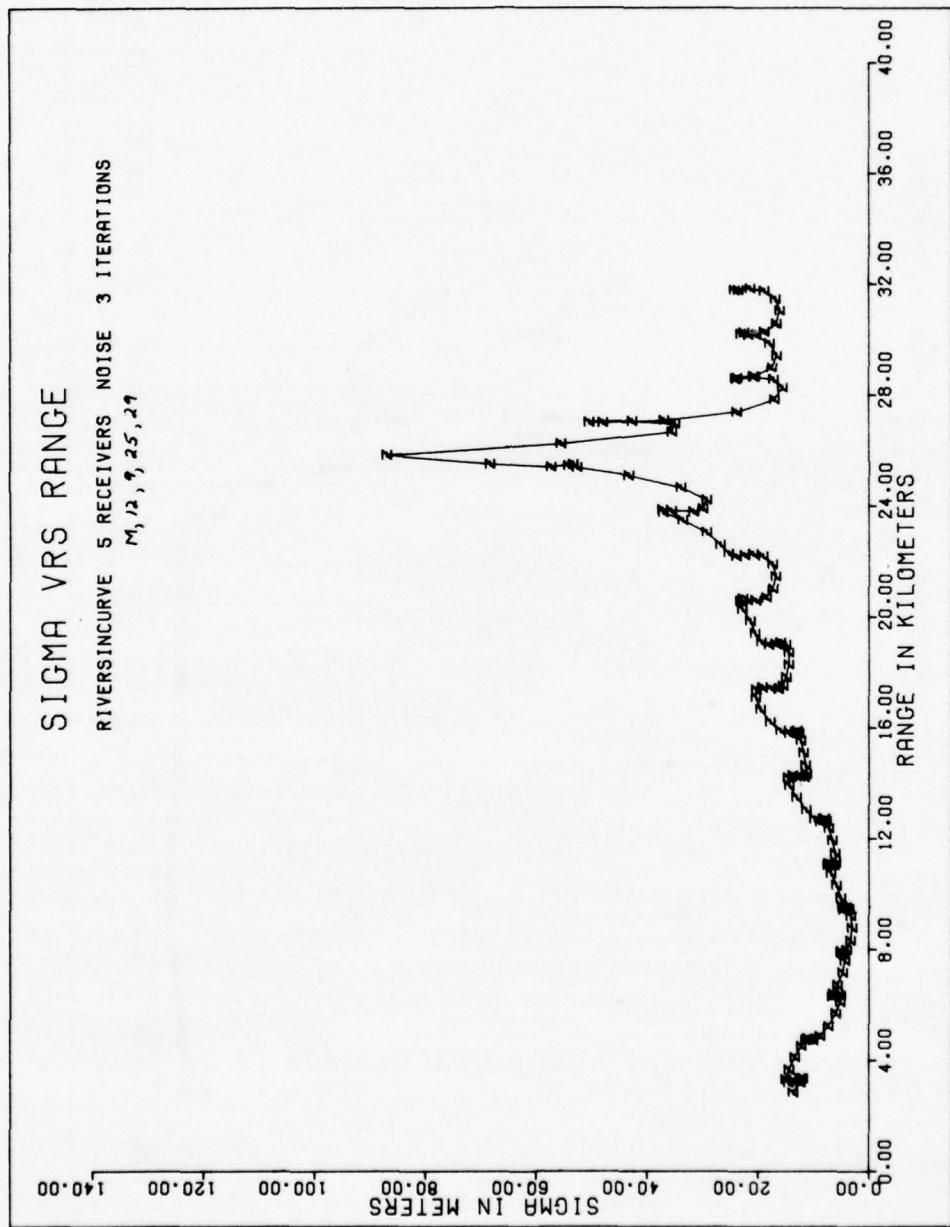
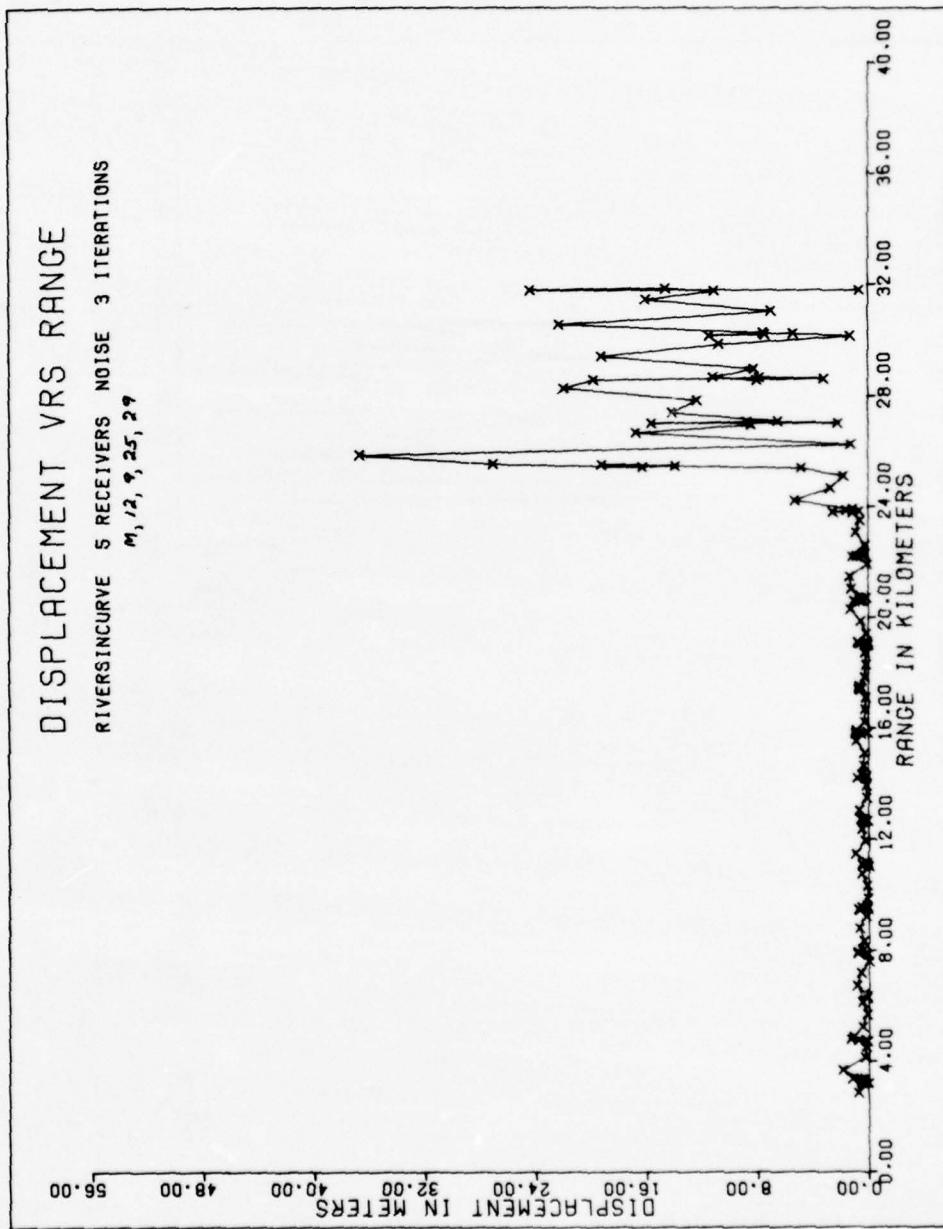


FIGURE 6
Z SIGMA VS. RANGE, SINE PATH



A-7

FIGURE 7
X DISPLACEMENT VS. RANGE, SINE PATH

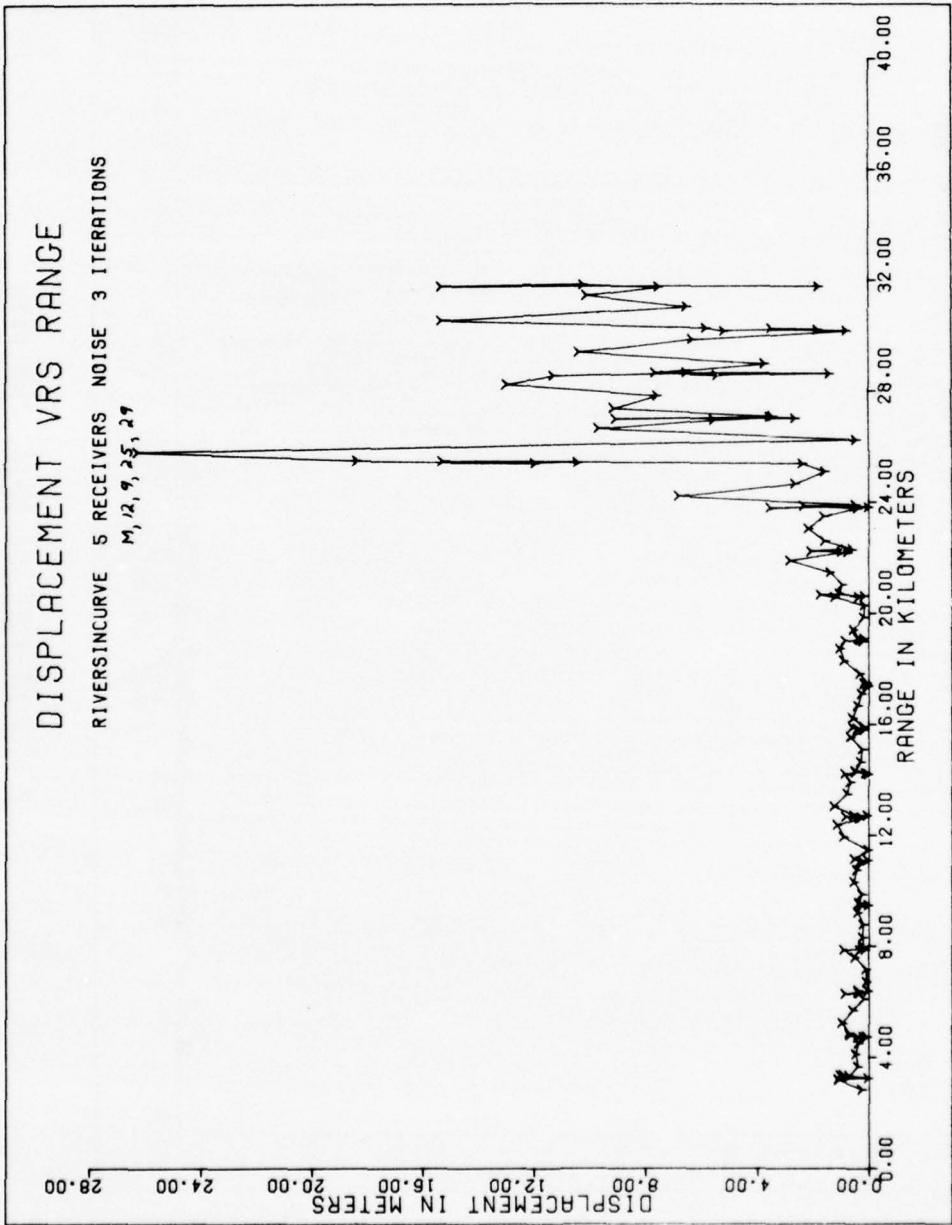
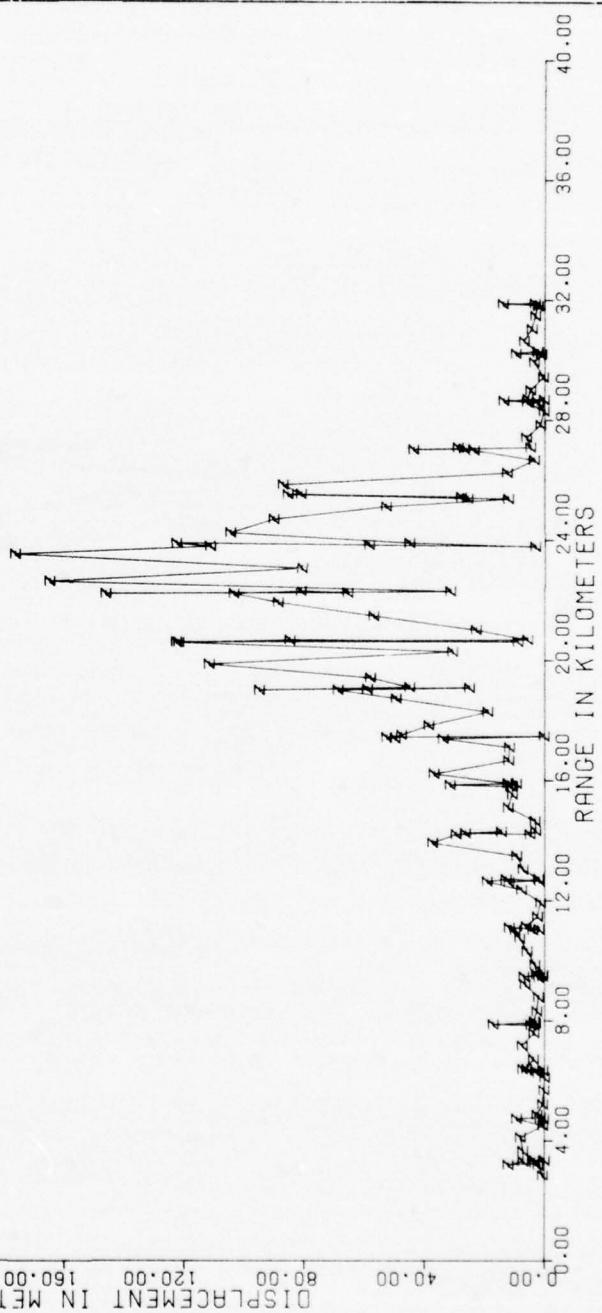


FIGURE 8
Y DISPLACEMENT VS. RANGE, SINE PATH

DISPLACEMENT VRS RANGE

RIVER S IN CURVE NOISE NONE ON MARYLAND SIDE 4 REC
M, 9, 25, 29

DISPLACEMENT IN METERS 0.00 40.00 80.00 120.00 160.00 200.00 240.00 280.00



A-9

FIGURE 9
Z DISPLACEMENT VS. RANGE, SINE PATH

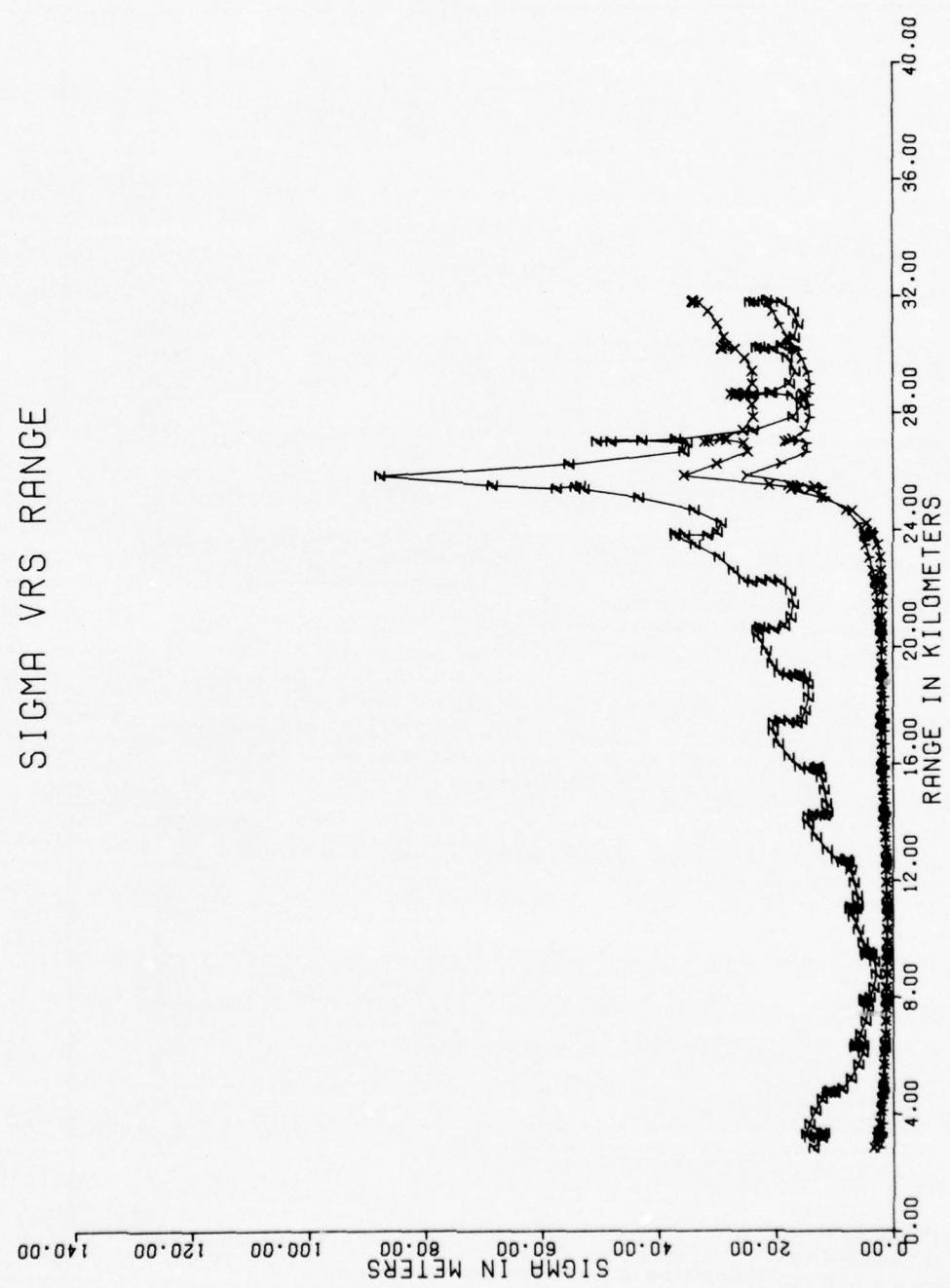


FIGURE 10
X, Y, Z SIGMA VS. RANGE, SINE PATH, 5 ITERATIONS

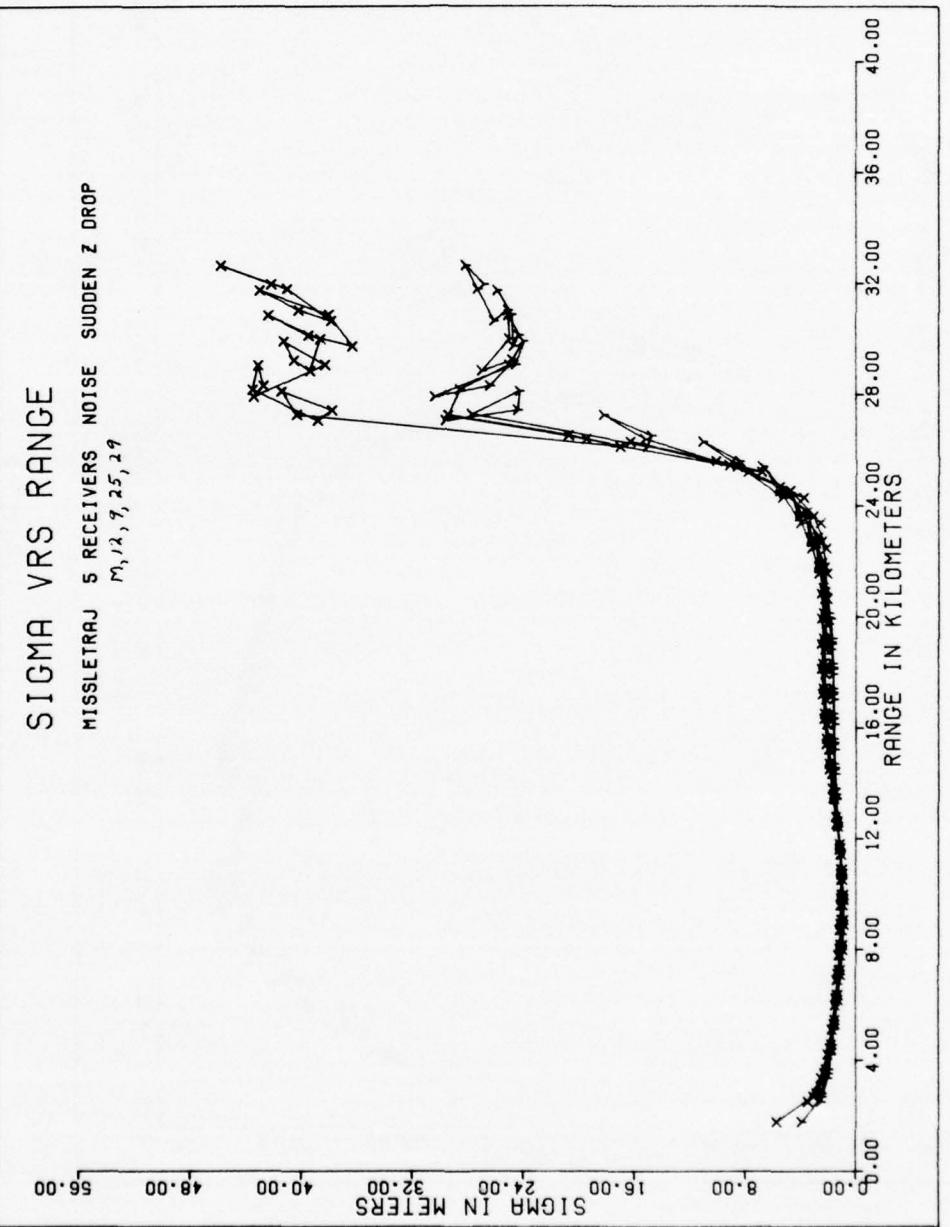


FIGURE 11
X AND Y SIGMA VS. RANGE, MISSILE PATH

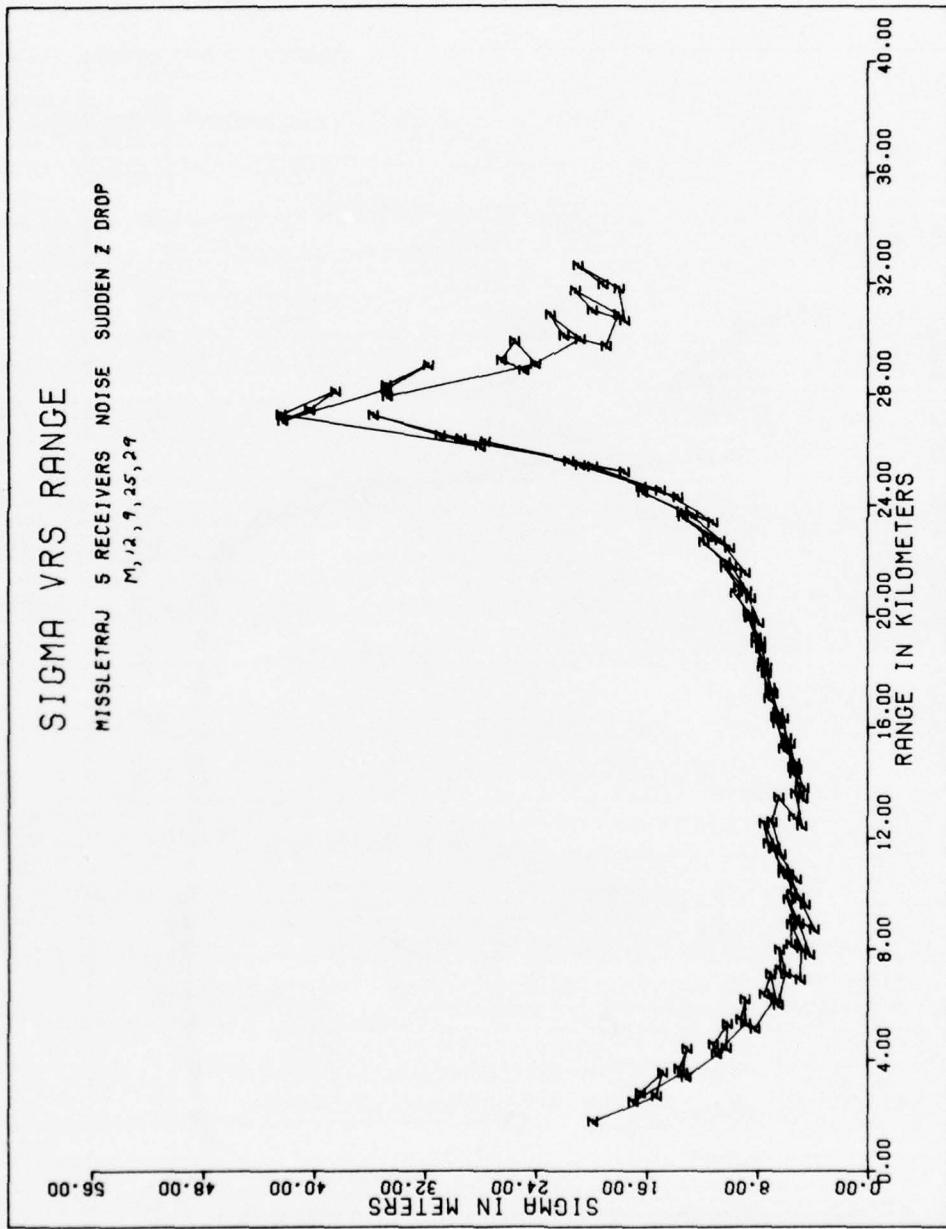


FIGURE 12
Z SIGMA VS. RANGE, MISSILE PATH

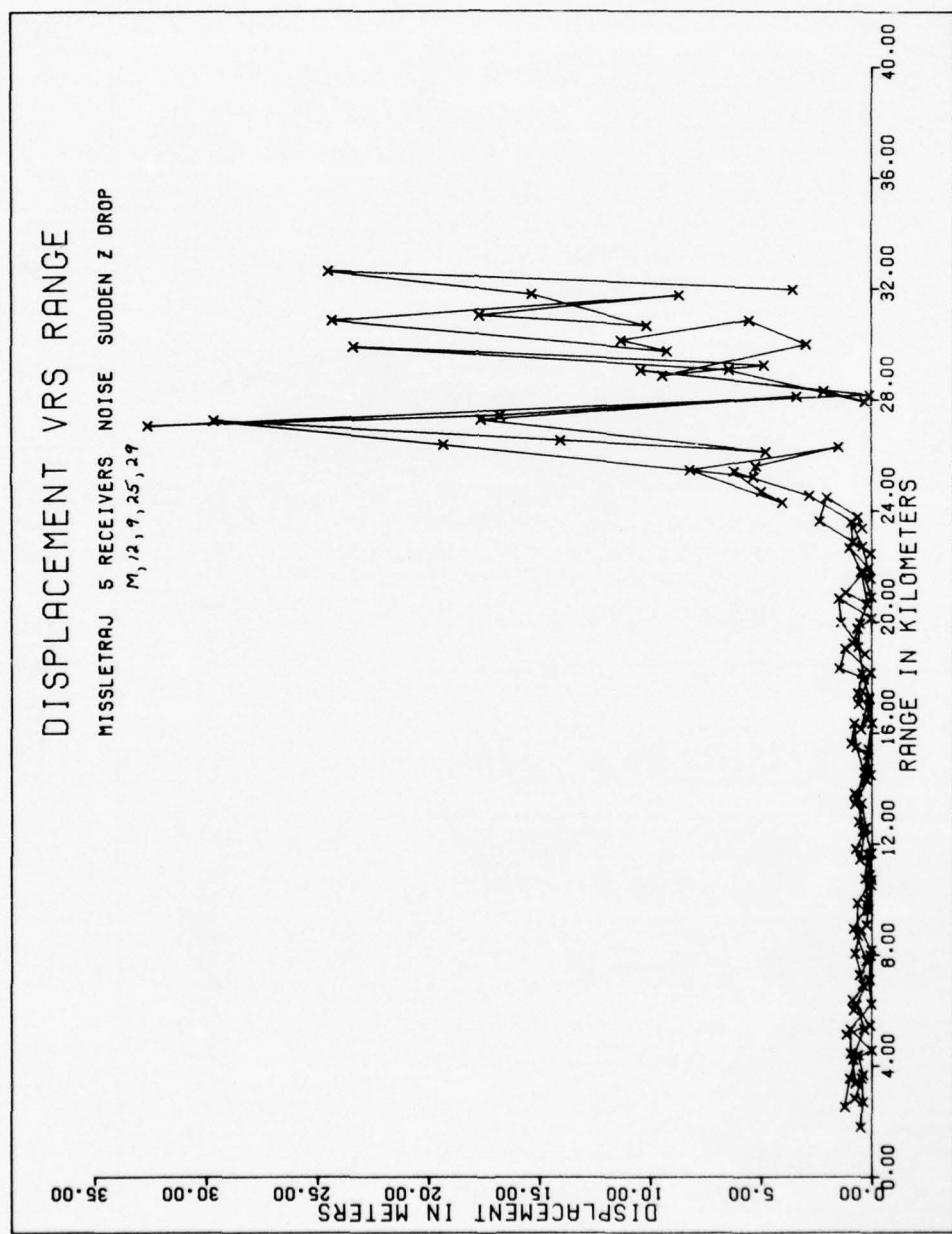


FIGURE 13
X DISPLACEMENT VS. RANGE, MISSILE PATH

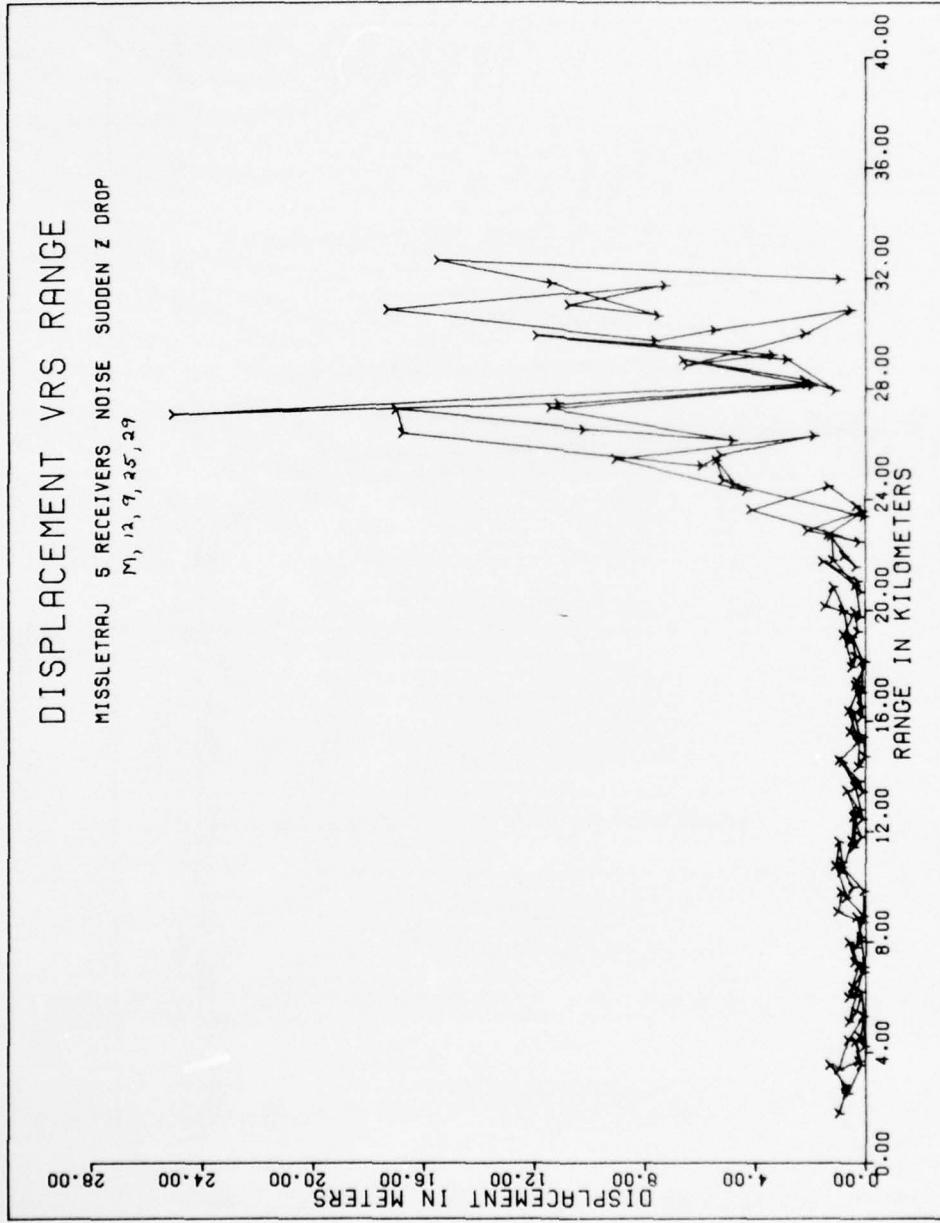


FIGURE 14
Y DISPLACEMENT VS. RANGE, MISSILE PATH

DISPLACEMENT VRS RANGE
MISSILETRAJ 5 RECEIVERS NOISE SUDDEN Z DROP
M, 12, 9, 25, 29

DISPLACEMENT IN METERS 0.00 8.00 16.00 24.00 32.00 40.00 48.00 56.00

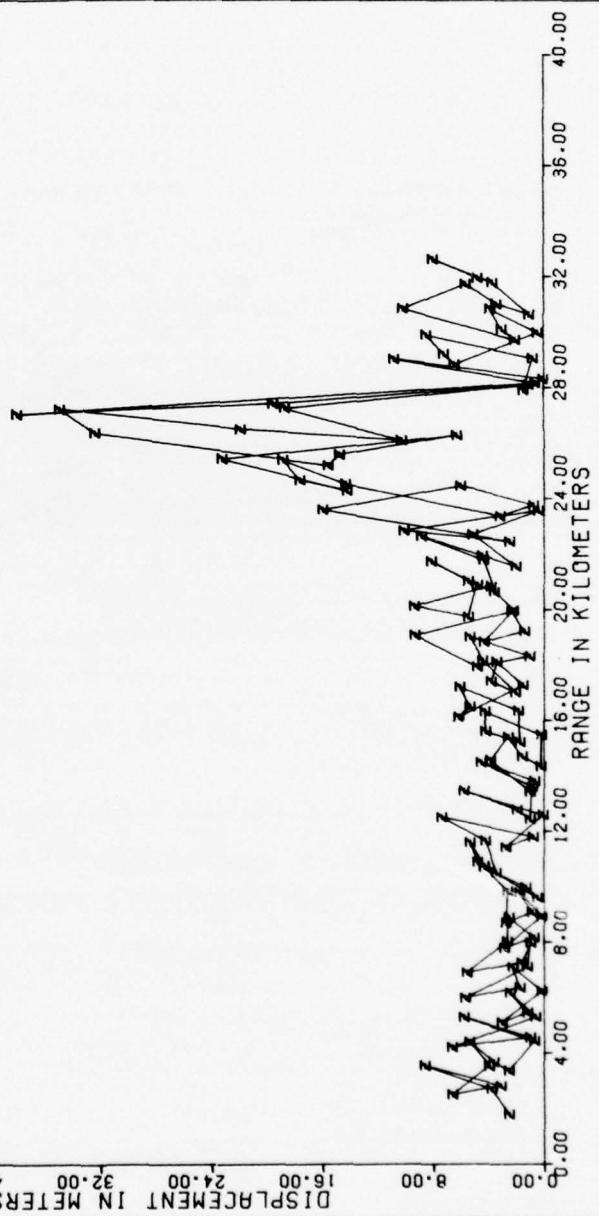
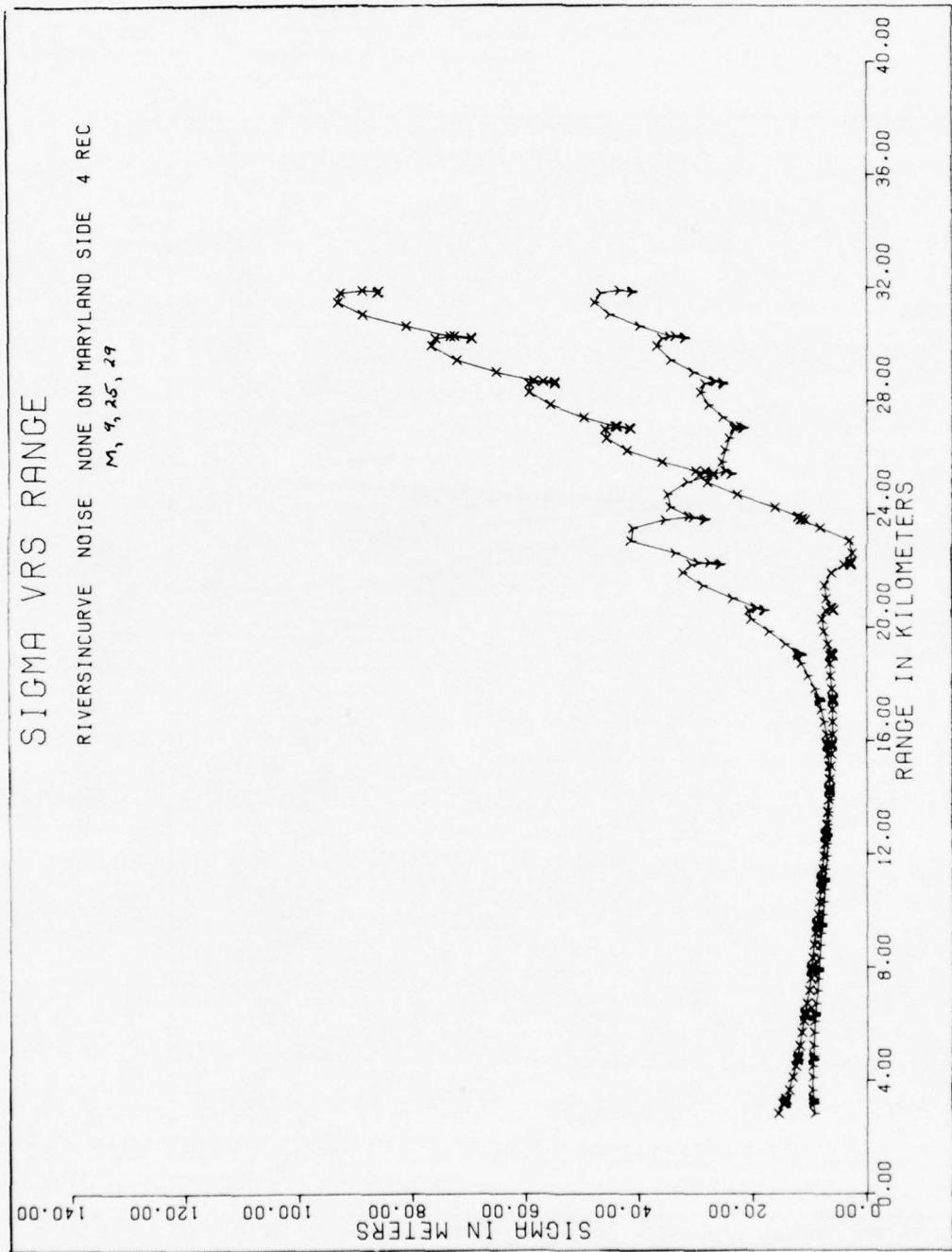
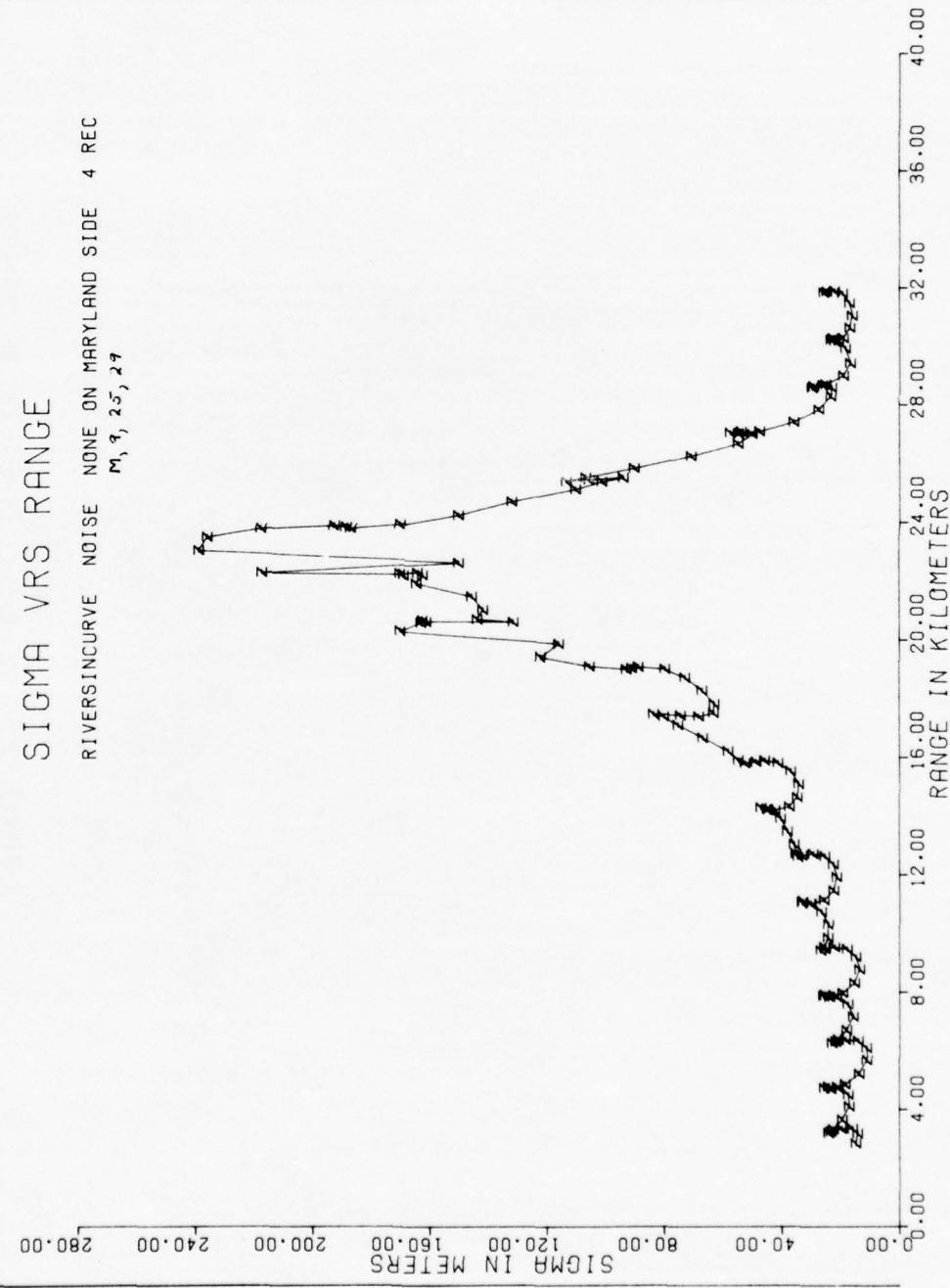


FIGURE 15
Z DISPLACEMENT VS. RANGE, MISSILE PATH



A-16

FIGURE 16
X AND Y SIGMA VS. RANGE, SINE PATH, NO MARYLAND



A-17

FIGURE 17
Z SIGMA VS. RANGE, SINE PATH, NO MARYLAND

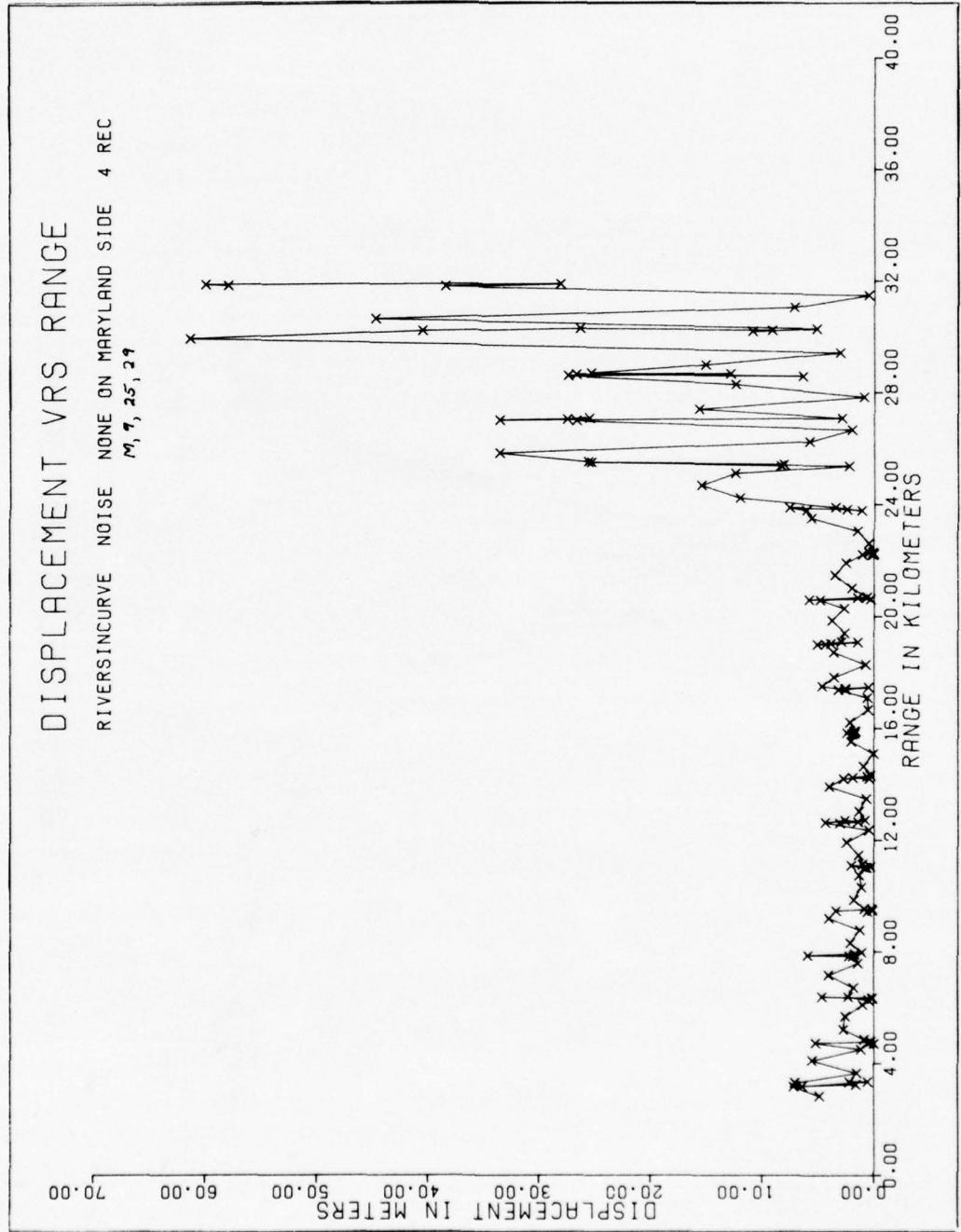


FIGURE 18
X DISPLACEMENT VS. RANGE, SINE PATH, NO MARYLAND

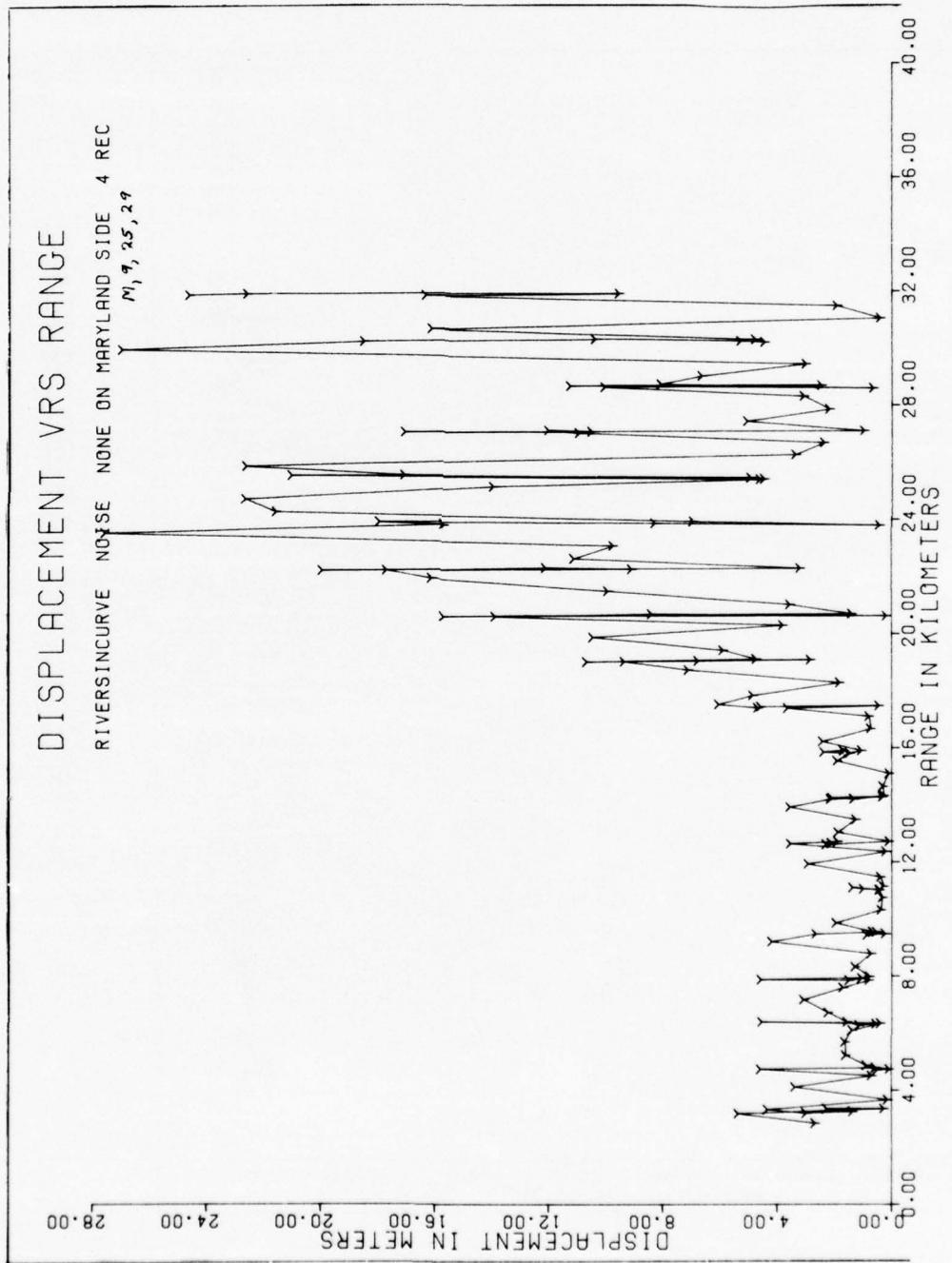
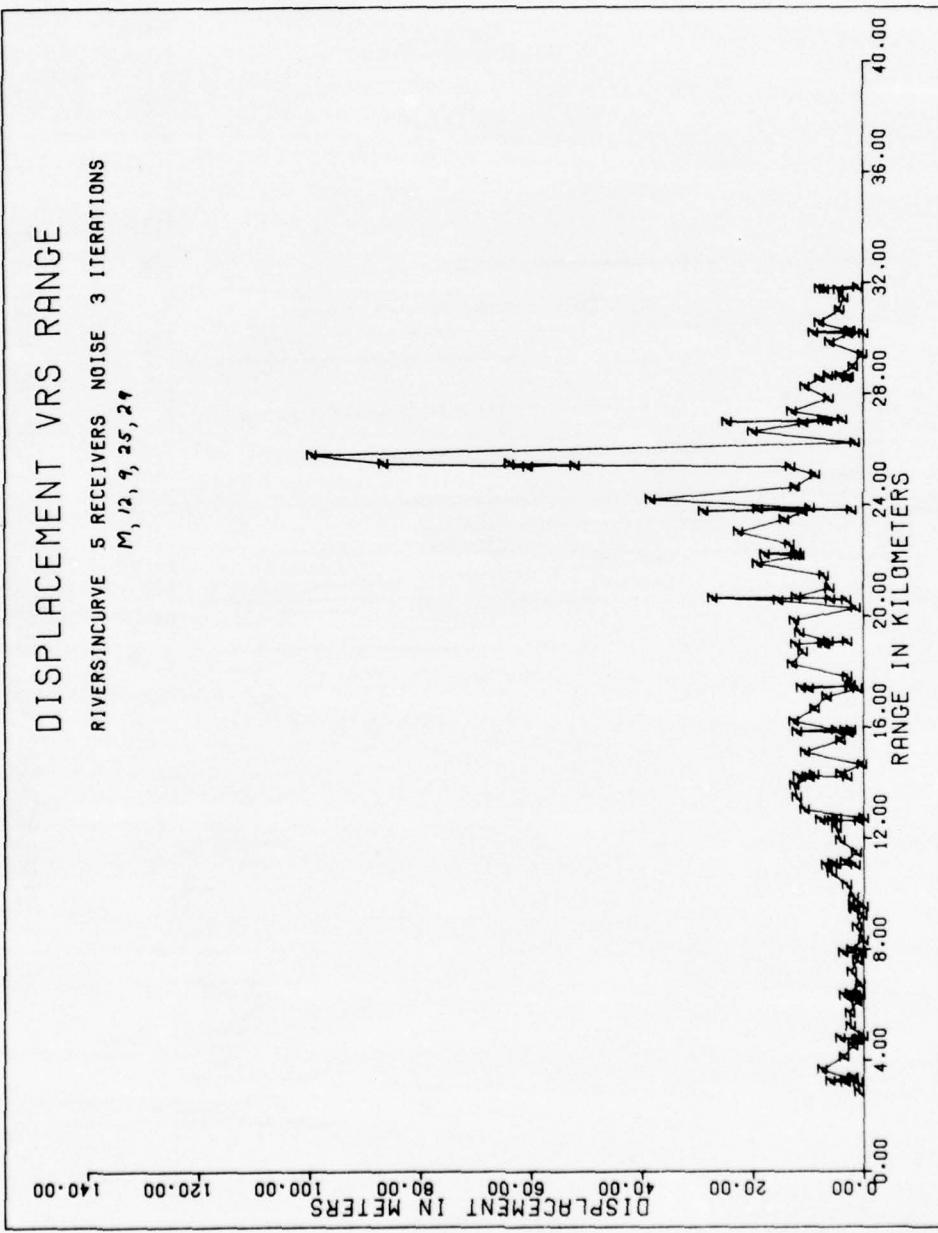


FIGURE 19
Y DISPLACEMENT VS. RANGE, SINE PATH, NO MARYLAND



A-20

FIGURE 20
Z DISPLACEMENT VS. RANGE, SINE PATH, NO MARYLAND

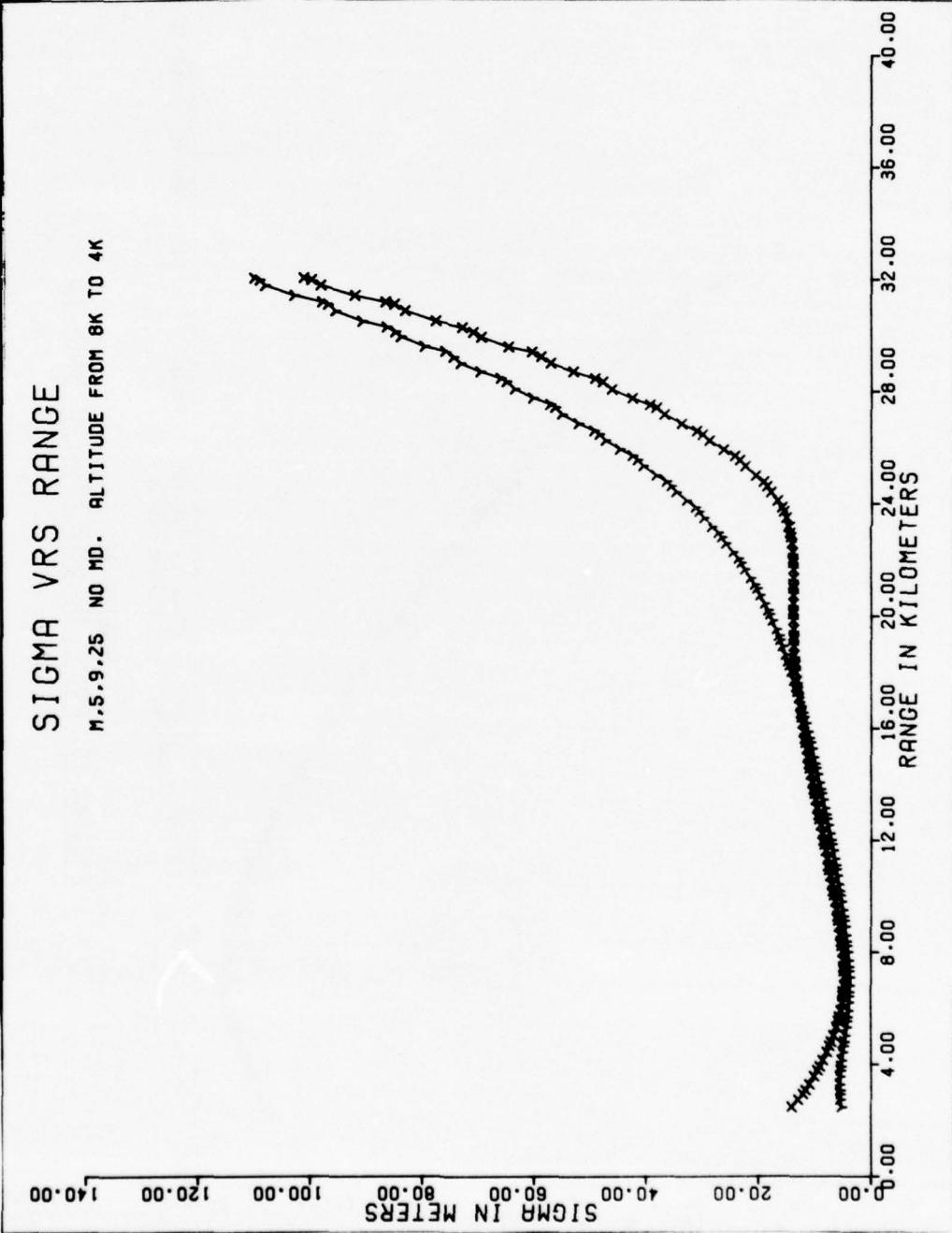


FIGURE 21
X AND Y SIGMA VS. RANGE, MISSILE PATH, NO MARYLAND

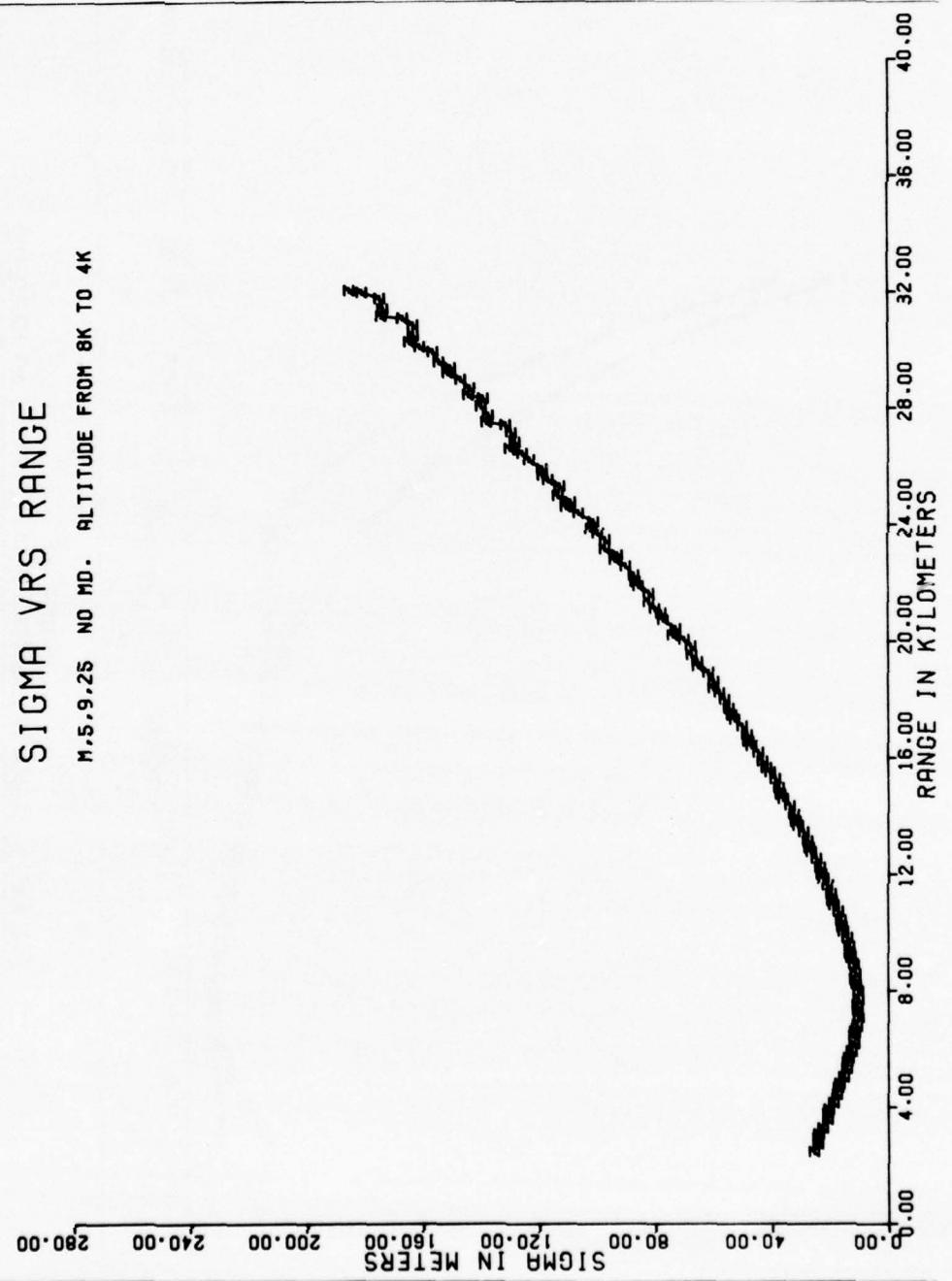


FIGURE 22
Z SIGMA VS. RANGE, MISSILE PATH, NO MARYLAND

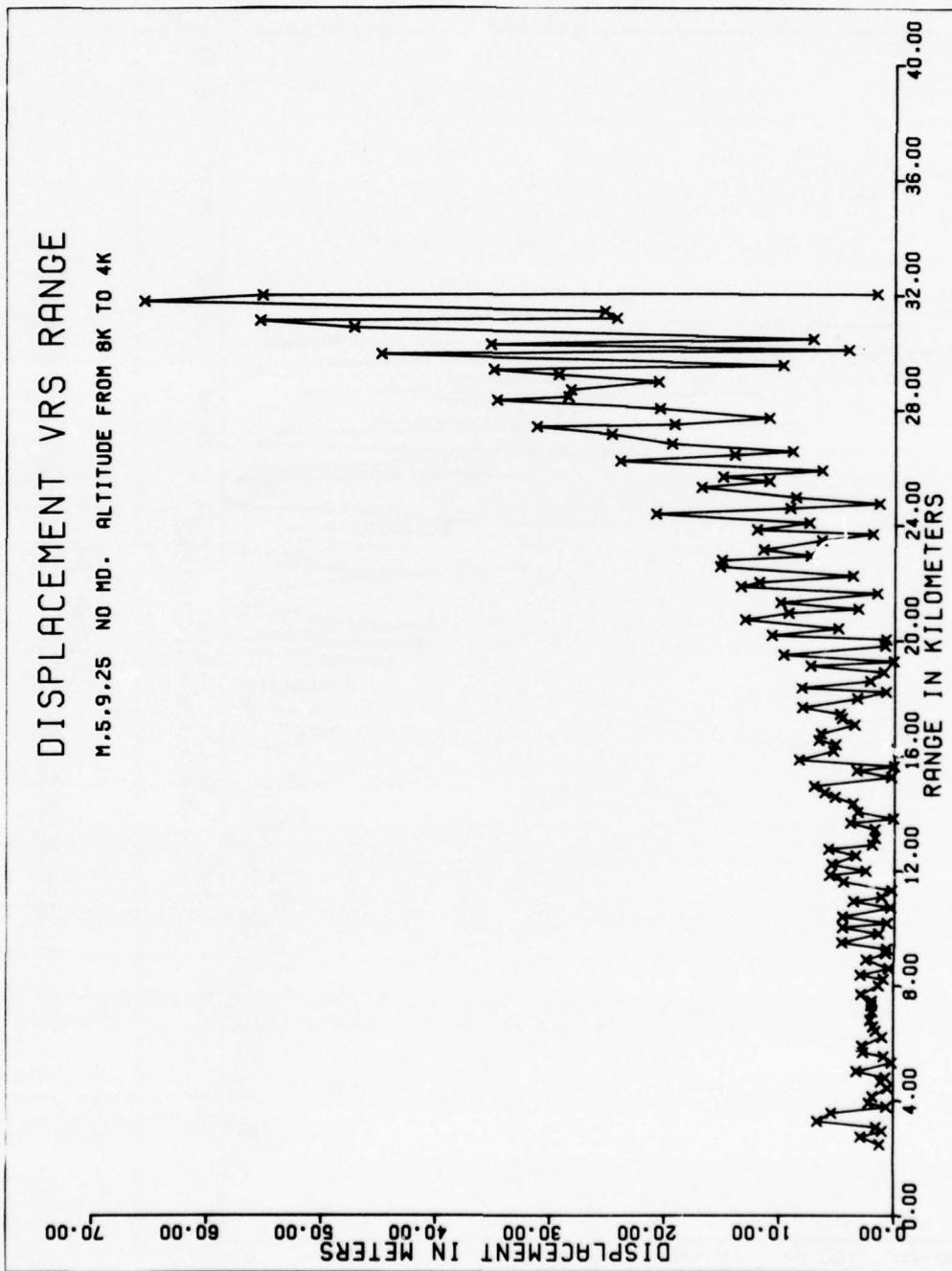


FIGURE 23
X DISPLACEMENT VS. RANGE, MISSILE PATH, NO MARYLAND

DISPLACEMENT VRS RANGE
M.5.9.25 NO MD. ALTITUDE FROM 8K TO 4K

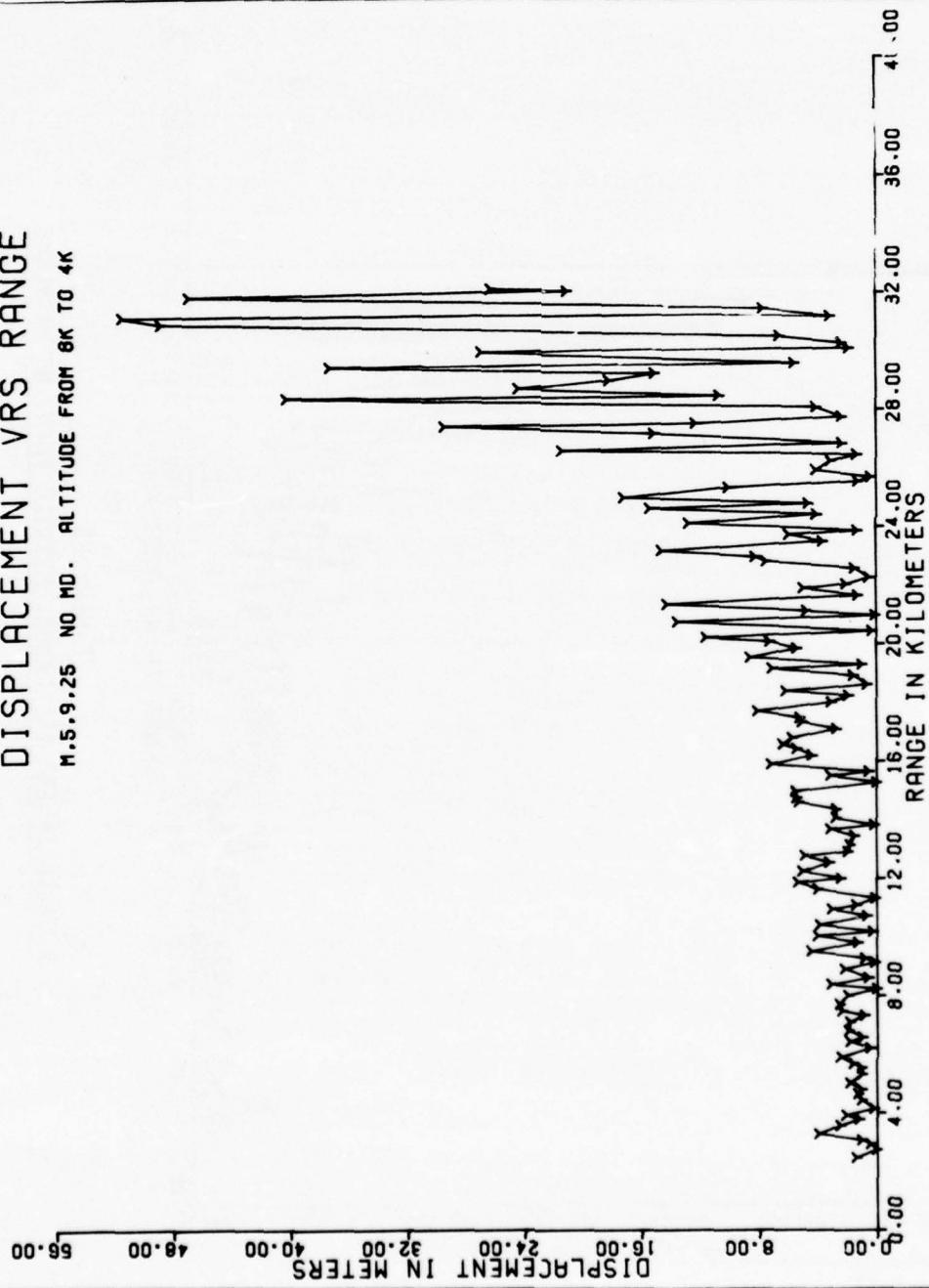
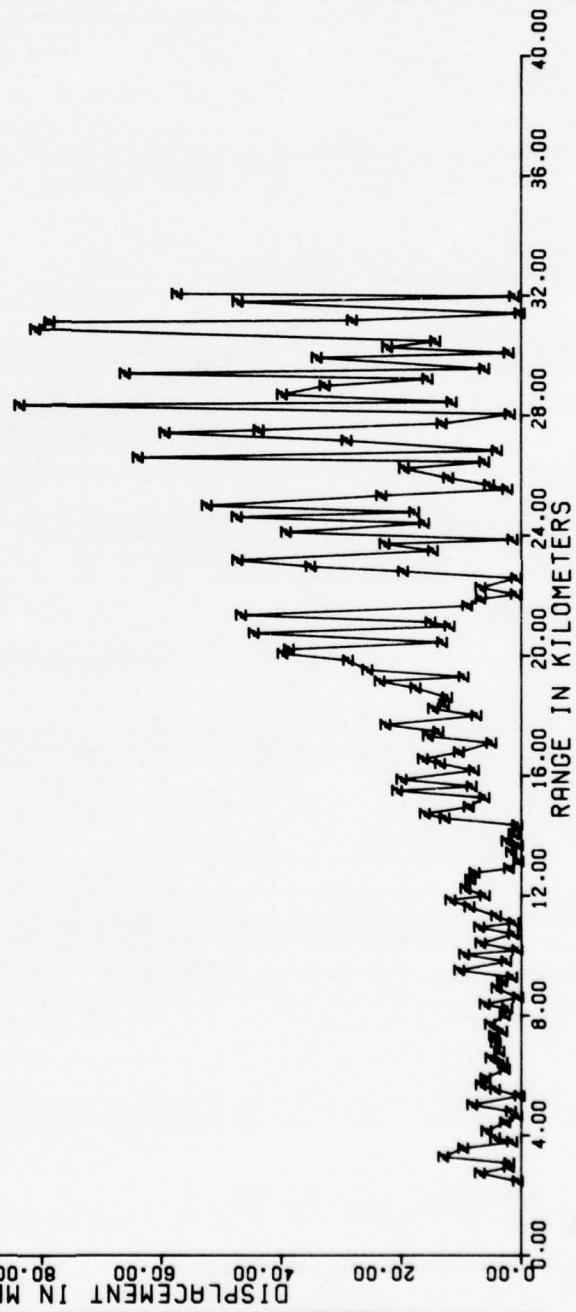


FIGURE 24
Y DISPLACEMENT VS. RANGE, MISSILE PATH, NO MARYLAND

DISPLACEMENT VRS RANGE

H.5.9.25 NO MD. ALTITUDE FROM 8K TO 4K

DISPLACEMENT IN METERS 0.00 20.00 40.00 60.00 80.00 100.00 120.00 140.00



A-25

FIGURE 25
Z DISPLACEMENT VS. RANGE, MISSILE PATH, NO MARYLAND

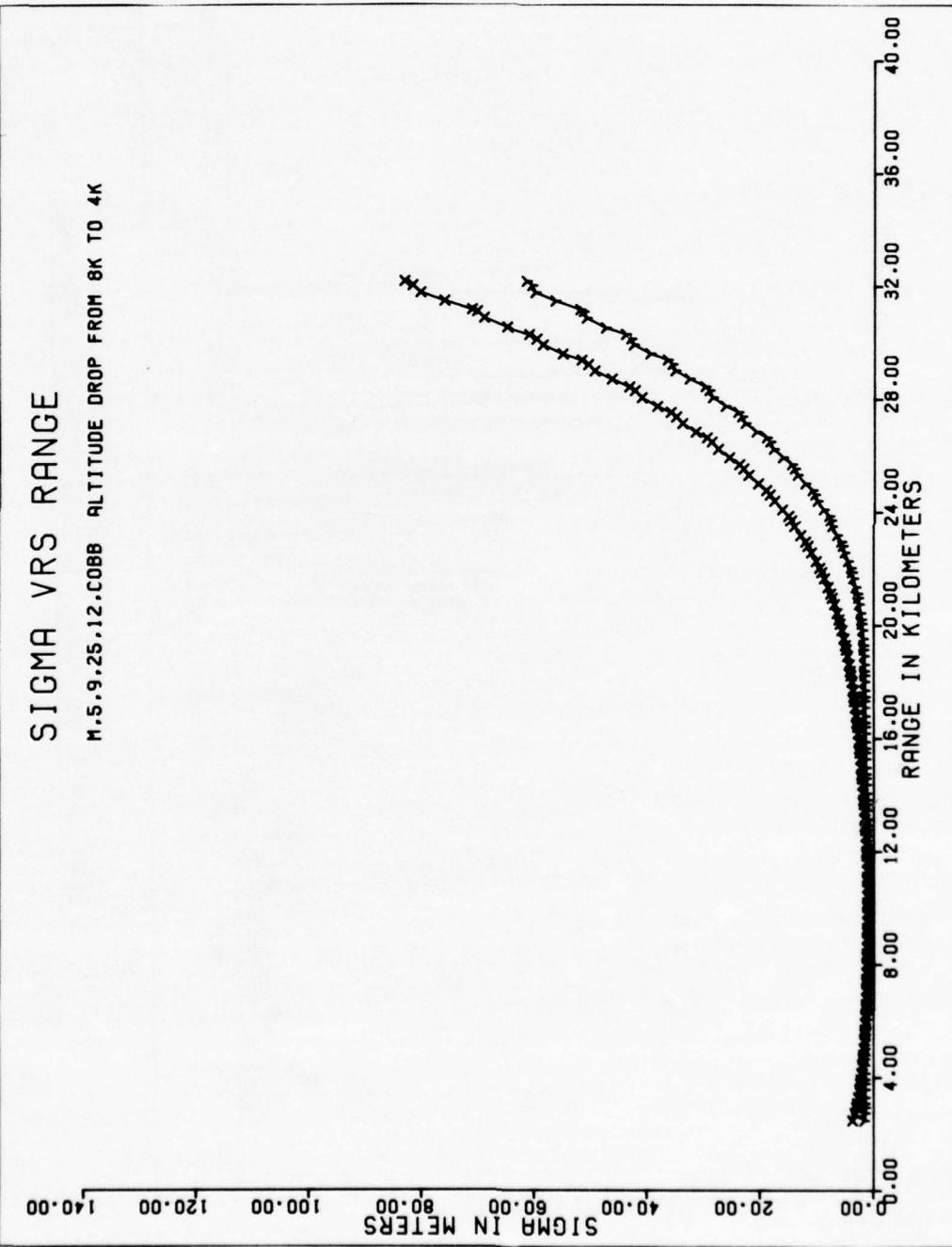


FIGURE 26
X AND Y SIGMA VS. RANGE, MISSILE PATH, MARYLAND SITES

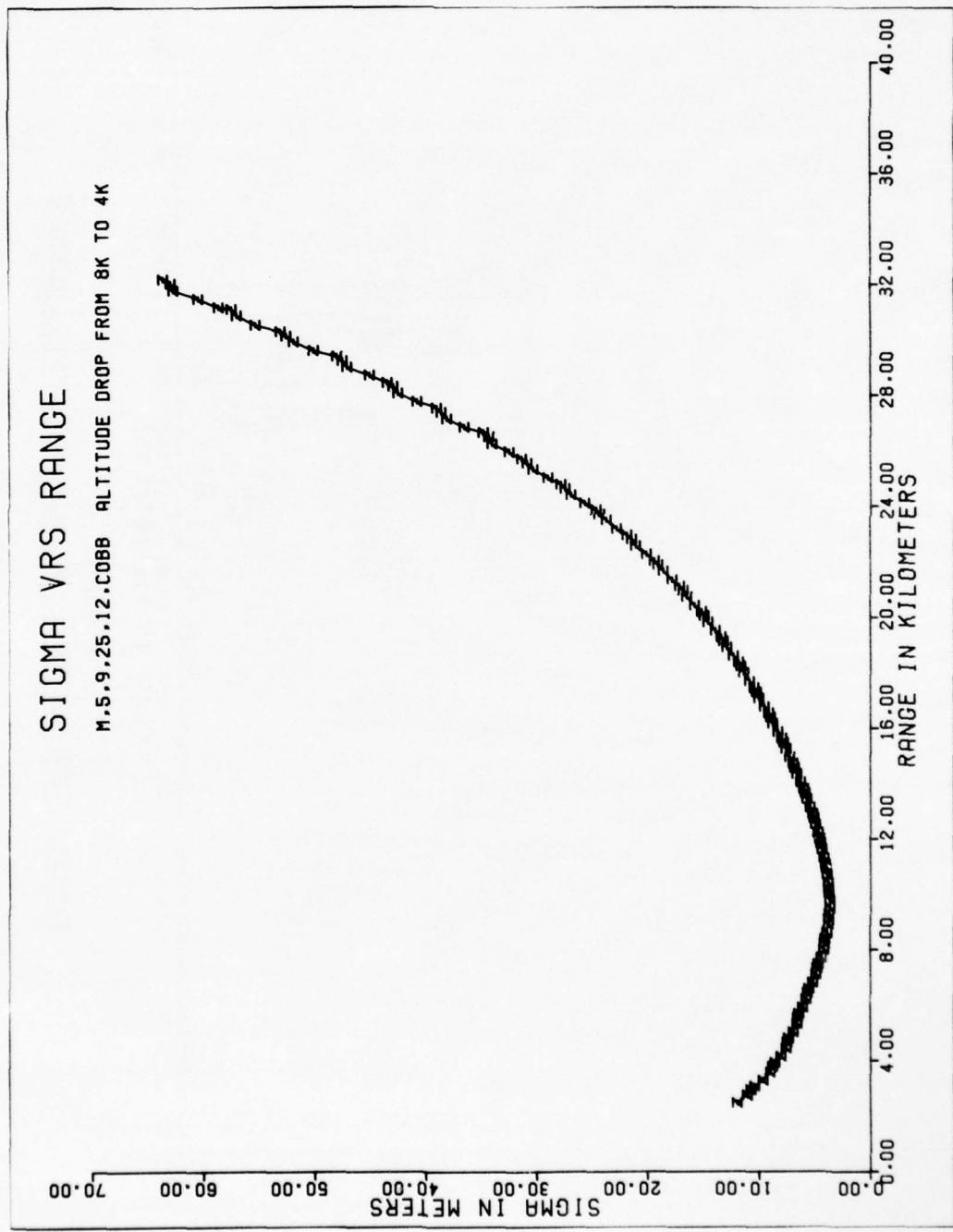


FIGURE 27
Z SIGMA VS. RANGE, MISSILE PATH, MARYLAND SITES

DISPLACEMENT VRS RANGE

M.5.9.25.12.COBB ALTITUDE DROP FROM 8K TO 4K

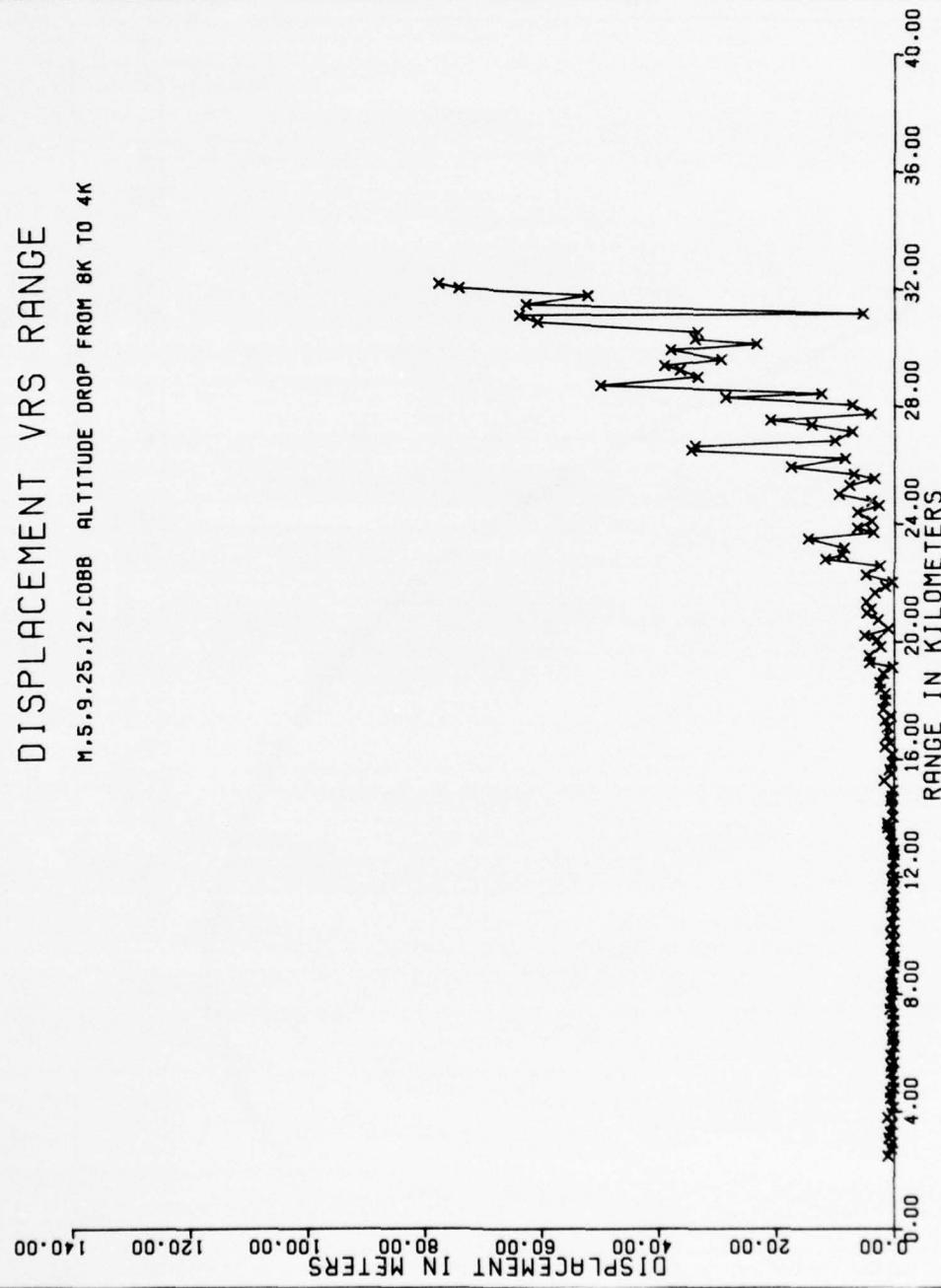


FIGURE 28
X DISPLACEMENT VS. RANGE, MISSILE PATH, MARYLAND SITES

DISPLACEMENT VRS RANGE
M.5.9.25.12.COBB ALTITUDE DROP FROM 8K TO 4K

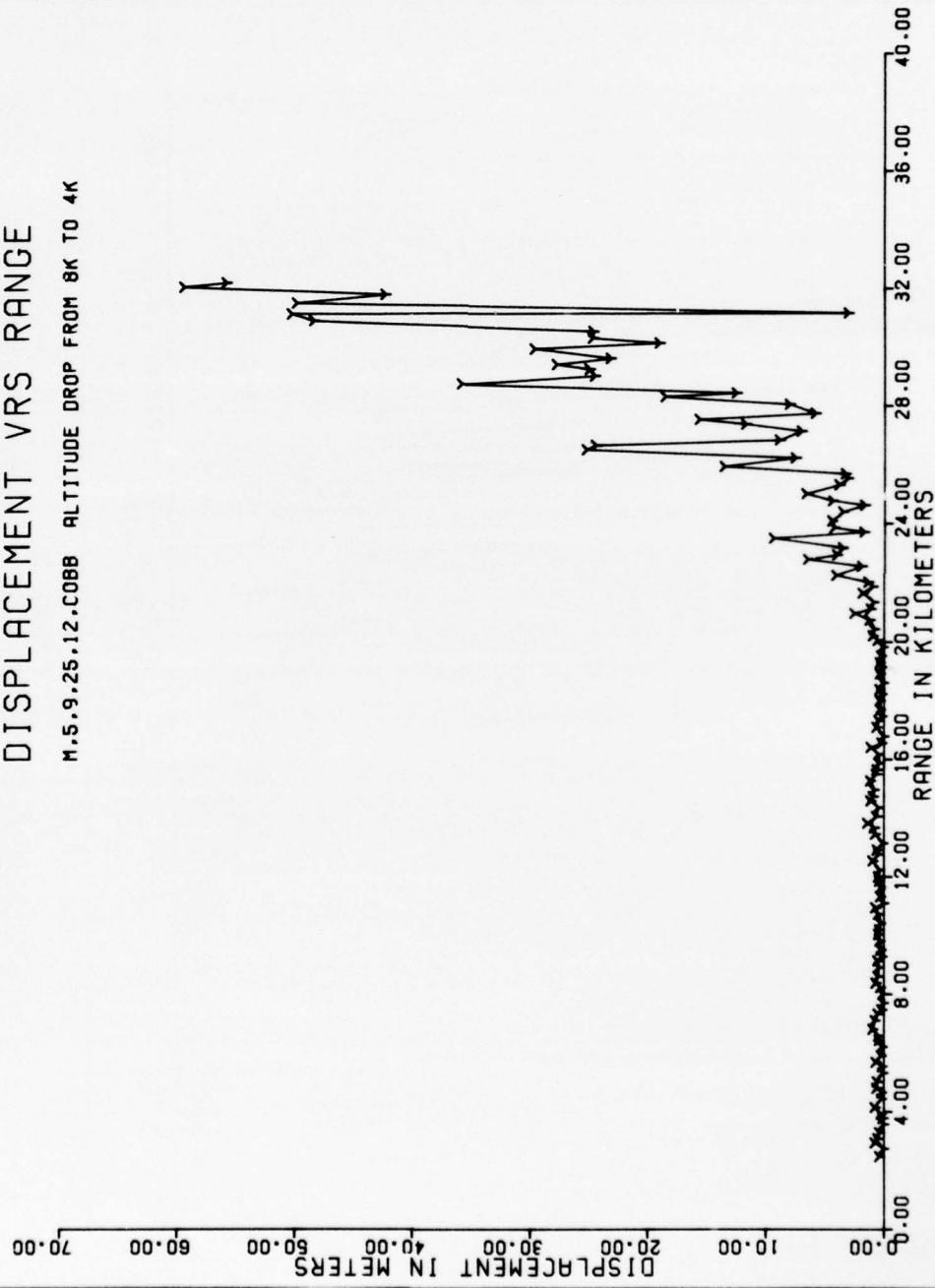


FIGURE 29
Y DISPLACEMENT VS. RANGE, MISSILE PATH, MARYLAND SITES

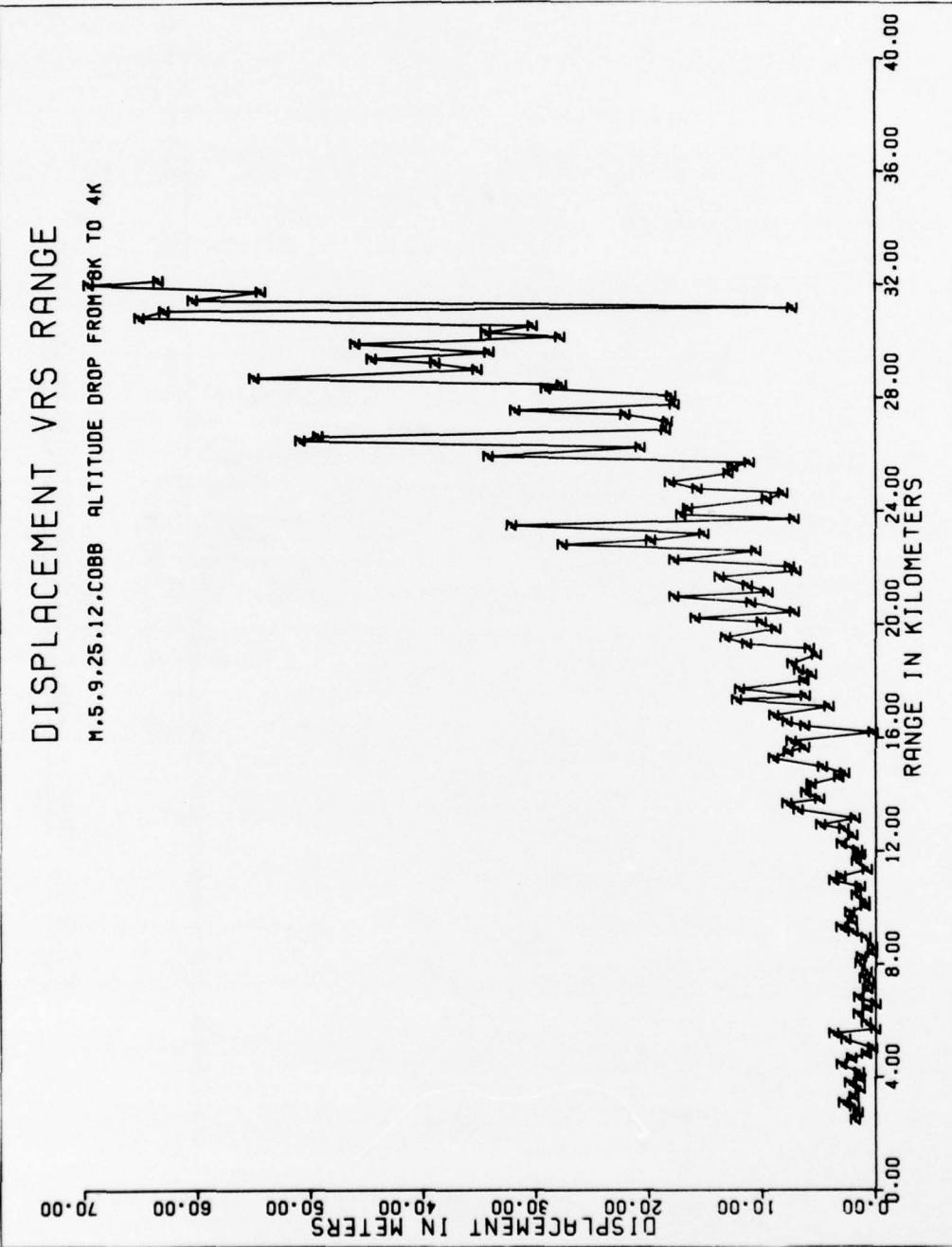


FIGURE 30
Z DISPLACEMENT VS. RANGE, MISSILE PATH, MARYLAND SITES

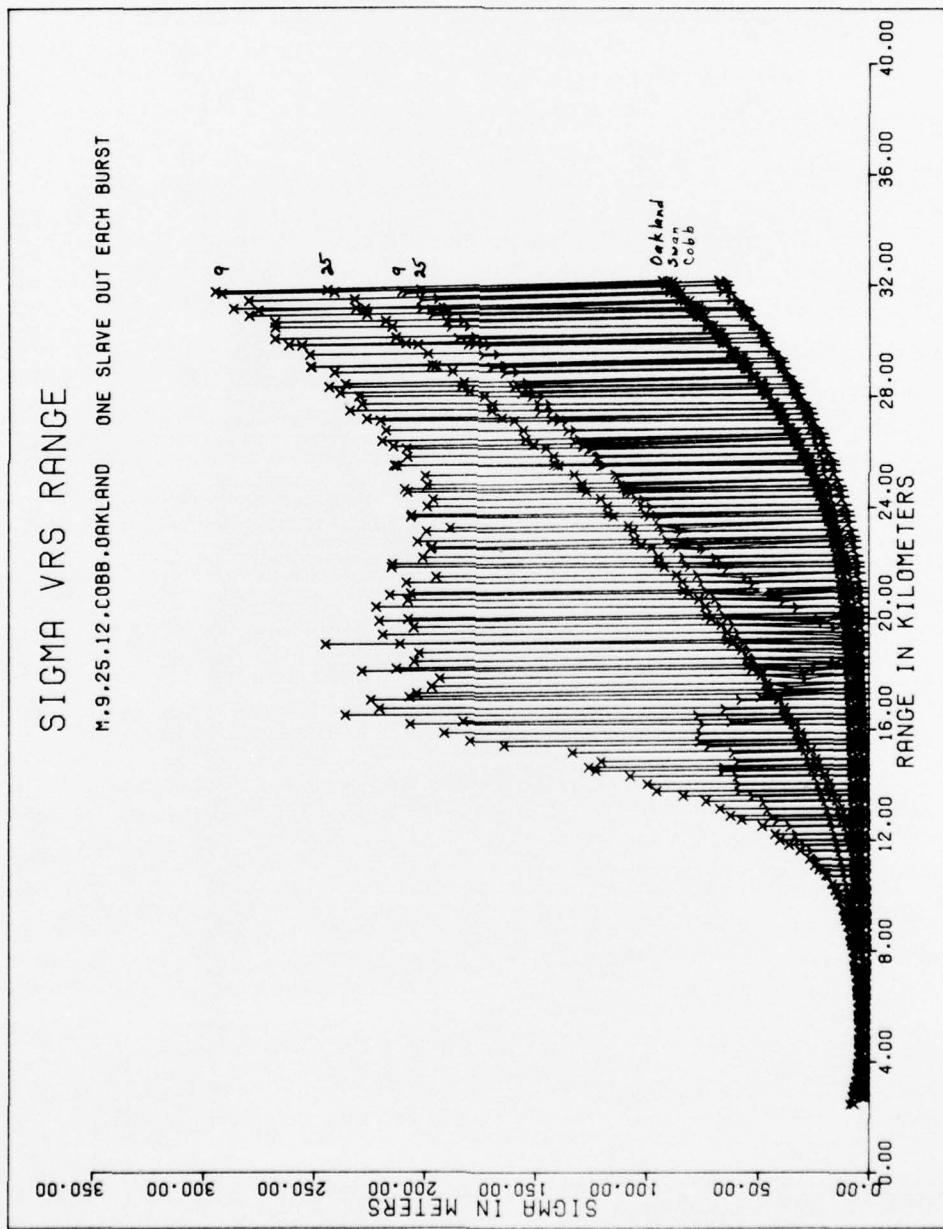


FIGURE 31
X AND Y SIGMA VS. RANGE, ONE SLAVE OUT (M,9,25,12,C,0)

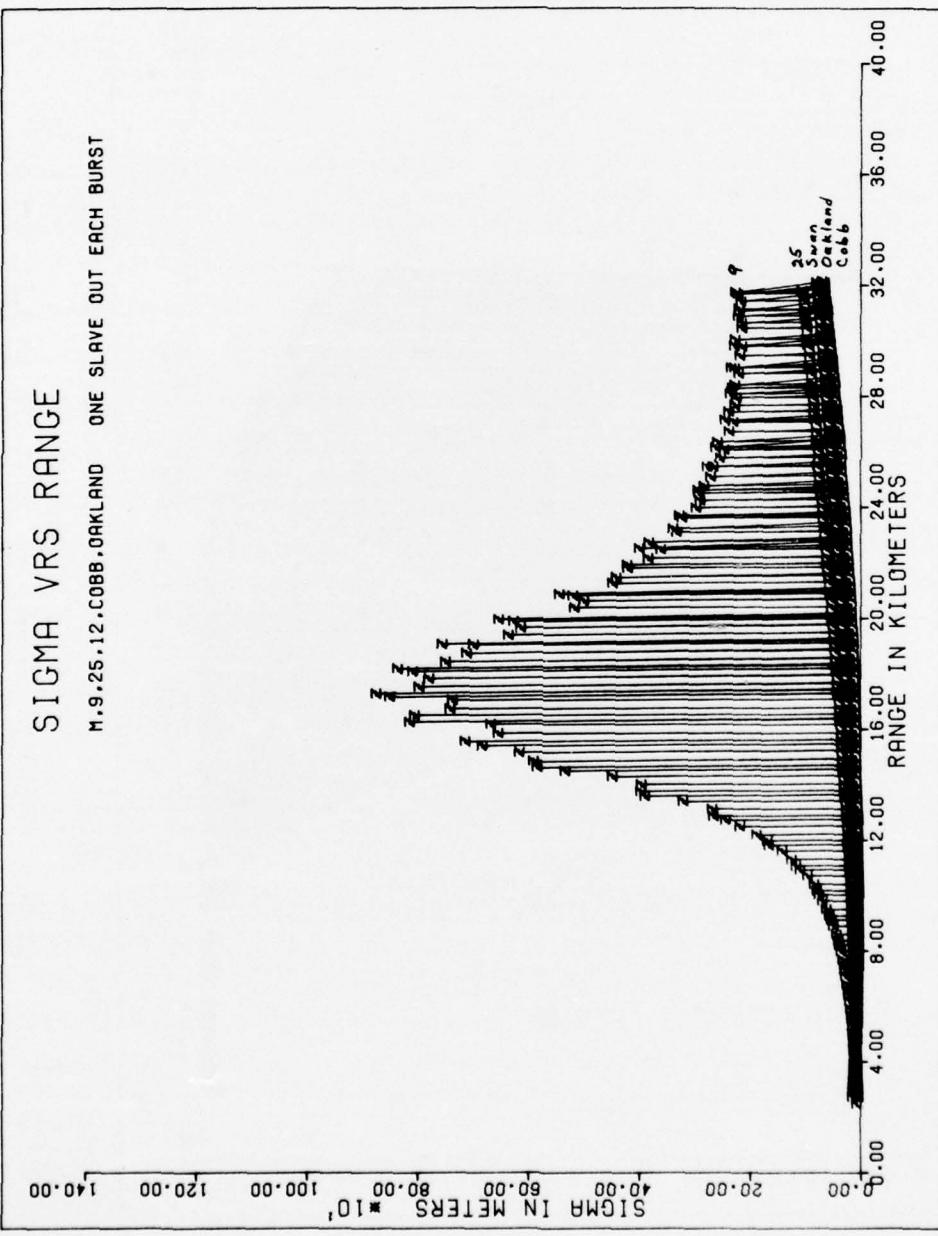


FIGURE 32
Z SIGMA VS RANGE, ONE SLAVE OUT (M,9,25,12,C,0)

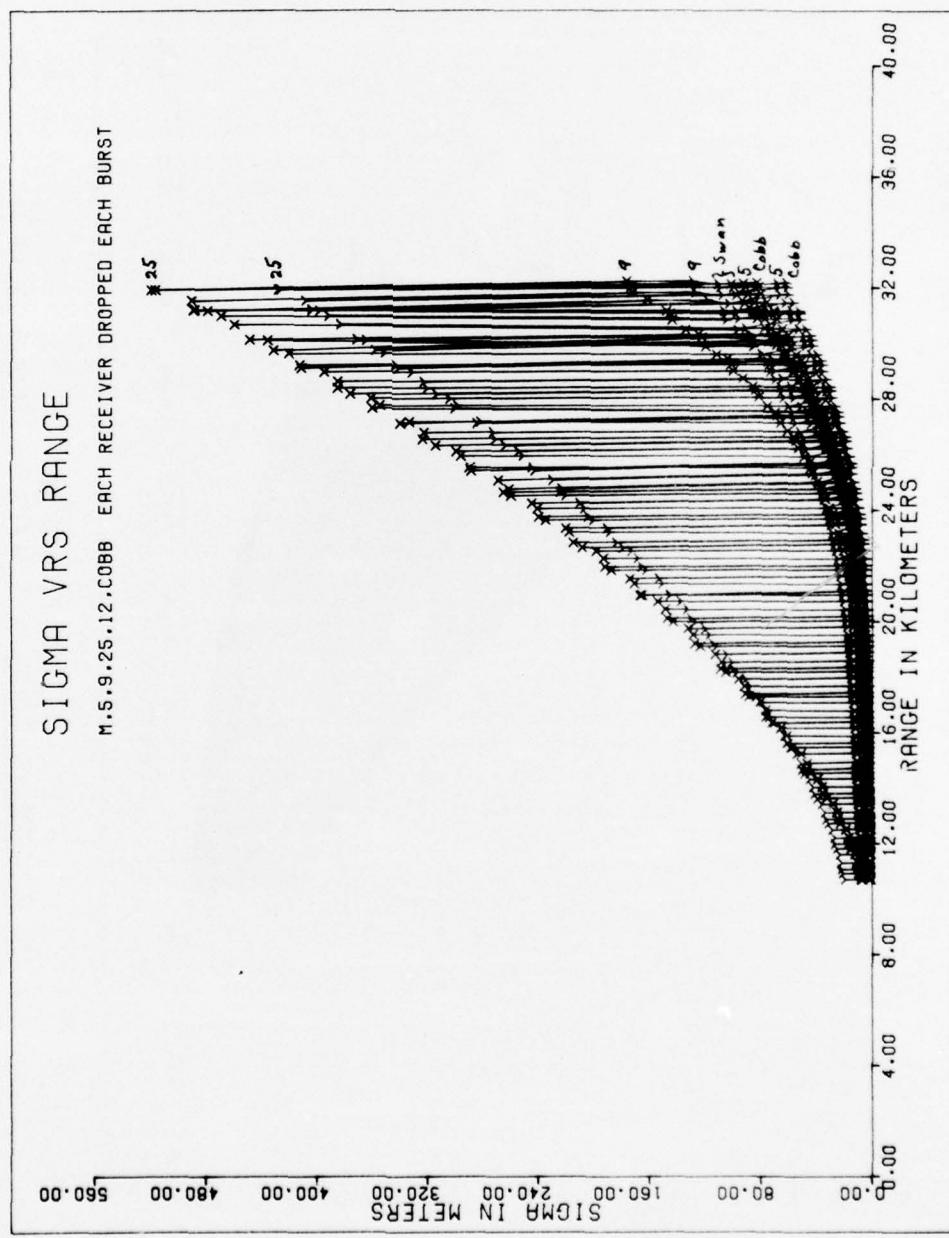


FIGURE 33
X AND Y SIGMA VS. RANGE, ONE SLAVE OUT (M,5,9,25,12,C)

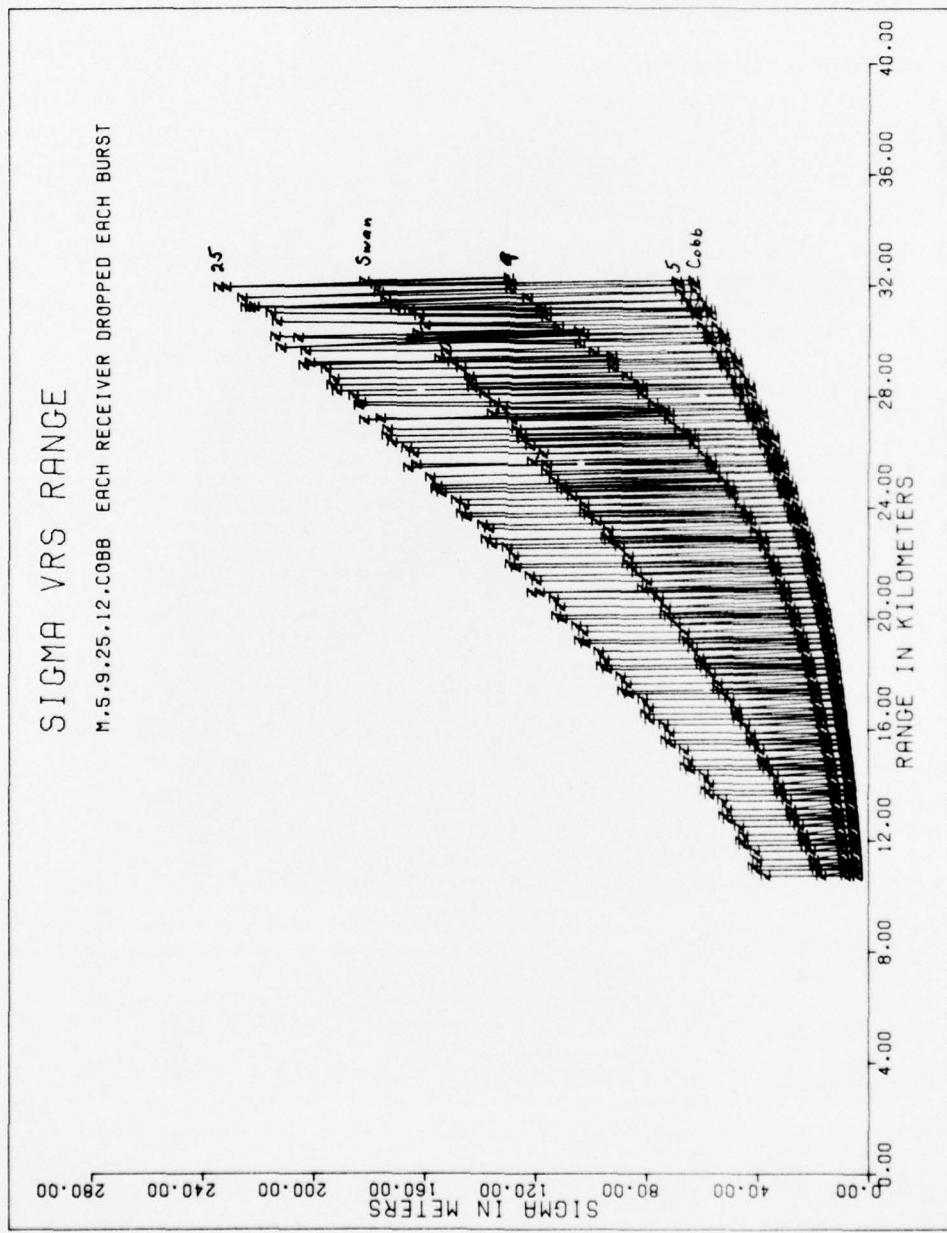


FIGURE 34
Z SIGMA VS. RANGE, ONE SLAVE OUT (M.5,9,25,12,C)

APPENDIX B
METHOD OF LEAST SQUARES

Assume, as in the discussion on page 2, we are able to resolve a quantity T . Let these measured values of T be denoted by T_{OBS} . Assume the quantity measured is a function of some parameter space $T = T(P_1, \dots, P_k)$. Of interest are the specific values of the parameter set P_1, \dots, P_k that relate to the measured values T_{OBS} .

Taking M measurements of T results in the following system of equations, relating measured values of T to the parameters P_1, \dots, P_k

$$(T_{OBS})_i = T_i(P_1, \dots, P_k) \text{ for } i = 1, 2, \dots, M \quad (1)$$

If $M > K$, the system is overdetermined, i.e. there are more equations than unknown parameters. In order to obtain a unique solution to Equation 1, an additional criterion, the least square condition, is imposed. This criterion is that

$$\sum_{i=1}^M \frac{[(T_{OBS})_i - T_i(P_1, \dots, P_k)]^2}{\sigma_i^2}$$

be a minimum. σ_i is the standard deviation of the i th measurement. Let (P_1^0, \dots, P_k^0) be an initial estimate to the desired (P_1, \dots, P_k) . Let

$$(T_{COMP})_i = T_i(P_1, \dots, P_k) \quad (2)$$

Expanding $T(P_1, \dots, P_k)$ in a Taylor Series around the point (P_1^0, \dots, P_k^0) results in:

$$T_i(P_1, \dots, P_k) = T_i(P_1^0, \dots, P_k^0) + \sum_{j=1}^k \frac{\partial T_i}{\partial P_j} (P_j - P_j^0) + \text{higher terms} \quad (3)$$

by neglecting the higher order terms and defining

$$\Delta P_j = (P_j - P_j^0) \quad (4)$$

the least squares condition is satisfied if for all parameters P_L ($L = 1, \dots, K$)

$$\frac{\partial}{\partial P_L} \sum_{i=1}^M \frac{[(T_{OBS})_i - (T_i(P_1^0, \dots, P_k^0) - \sum_{j=1}^k \frac{\partial T_i}{\partial P_j} \Delta P_j)]^2}{\sigma_i^2} = 0 \quad (5)$$

Let $(O-C)_i = (TOBS)_i - T_i(p_1, \dots, p_k)$. Then

$$\sum_{i=1}^M \frac{(O-C)_i^2}{\sigma_i^2} \frac{\partial T_i}{\partial p_L} = \sum_{j=1}^k \sum_{i=1}^M \frac{1}{\sigma_i^2} \frac{\partial T_i}{\partial p_j} \frac{\partial T_i}{\partial p_L} \Delta p_j \quad (6)$$

There is one such expression for each parameter p_L . Once the initial guess (p_1^0, \dots, p_k^0) is given, all quantities in the above expression are known except for Δp_j . Solving for Δp_j results in what is termed an improvement to the j th parameter.

The above equations can be written in matrix form as follows:

$$A_{ik} = \frac{1}{\sigma_L} \frac{\partial T_i}{\partial p_k}$$

$$B_{ik} = \sum_j \frac{1}{\sigma_i^2} \frac{\partial T_i}{\partial p_j} \frac{\partial T_i}{\partial p_k}$$

Note that B is symmetric and that $B = A^T A$. Equation 6 becomes

$$\sum_i \frac{(O-C)_i^2}{\sigma_i} A_{ik} = \sum_j B_{kj} \Delta p_j.$$

Let $E_k = \sum_i \frac{(O-C)_i^2}{\sigma_i} A_{ik}$,

and

$$\Delta P = \begin{pmatrix} \Delta p_1 \\ \vdots \\ \Delta p_k \end{pmatrix}.$$

Then

$$B \Delta P = E \quad (7)$$

Solution of the normal equations (7) results in ΔP or the improvements to the parameter set (p_1^0, \dots, p_k^0) . If the initial estimate (p_1^0, \dots, p_k^0) is sufficiently good, defining

$$(p_1, \dots, p_k) = (p_1^0 + \Delta p_1, \dots, p_k^0 + \Delta p_k)$$

results in the best (in the least squares sense) solution to the minimization of

$$\sum_{i=1}^N \frac{[(T_{OBS})_i - T_i(p_1, \dots, p_n)]^2}{\sigma_i^2}$$

Because we have made a linear approximation to $T(p_1, \dots, p_k)$, we have minimized a linear problem. Since in general $T - T(p_1, \dots, p_k)$ is not linear, it is necessary to iterate on this procedure until a best solution is determined.

In order to achieve an over-determined system, we make use of a-priori knowledge of the standard deviation of some of the parameters. The uncertainties in the RTT are determined by independent calibrations. Thus, after we form the system

$$B\Delta P = E,$$

to B we add the diagonal matrix

$$\frac{1}{\sigma_1^2}$$

$$\frac{1}{\sigma_2^2}$$

$$\frac{1}{\sigma_n^2}$$

where σ_i is the standard deviation of the ith parameter. The above diagonal matrix contains n non-zero terms. σ_x , σ_y , σ_z (the standard deviation of the position of the beacon transmitter) is assigned an infinite "sigma" and

$$\frac{1}{\sigma_x^2} \quad \text{and} \quad \frac{1}{\sigma_y^2} \quad \text{and} \quad \frac{1}{\sigma_z^2} \quad \text{are all zero.}$$

Now we have (with n receivers) $3 + n$ parameters, $n - 1$ observations, and n a-priori observations. For $r > 4$, $n-1+n>3+n$ and the system is over-determined.

The above diagonal matrix (i.e. the matrix of a-priori estimates

on RTT's) is added to the left-hand side of the normal equations only once. This one time occurs at the first point at which the normal equations are formed.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC/DL TR-3514	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) BEACON TRACKING SYSTEMS SIMULATION		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) ELODIE S. COLQUITT	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center Dahlgren Laboratory Dahlgren, Virginia 22448		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE June 1976
		13. NUMBER OF PAGES 52
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Beacon Tracking Systems		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The Beacon Tracking System is designed to determine the position of a beacon transmitter carried aboard a surface or air target using the Time-Difference-of-Arrival (TDOA) concept. Target position is computed based on a weighted least-squares solution of the TDOA data. A computer simulation of the system is used to investigate the system's sensitivity to several parameters. Sensitive parameters are shown to be: (1) receiver geometry relative to the target; (2) receiver position survey accuracy; (3) and loss of signal by one or more receivers during a track.		